UNIT I

CIRCUITS AND TRANSFORMERS

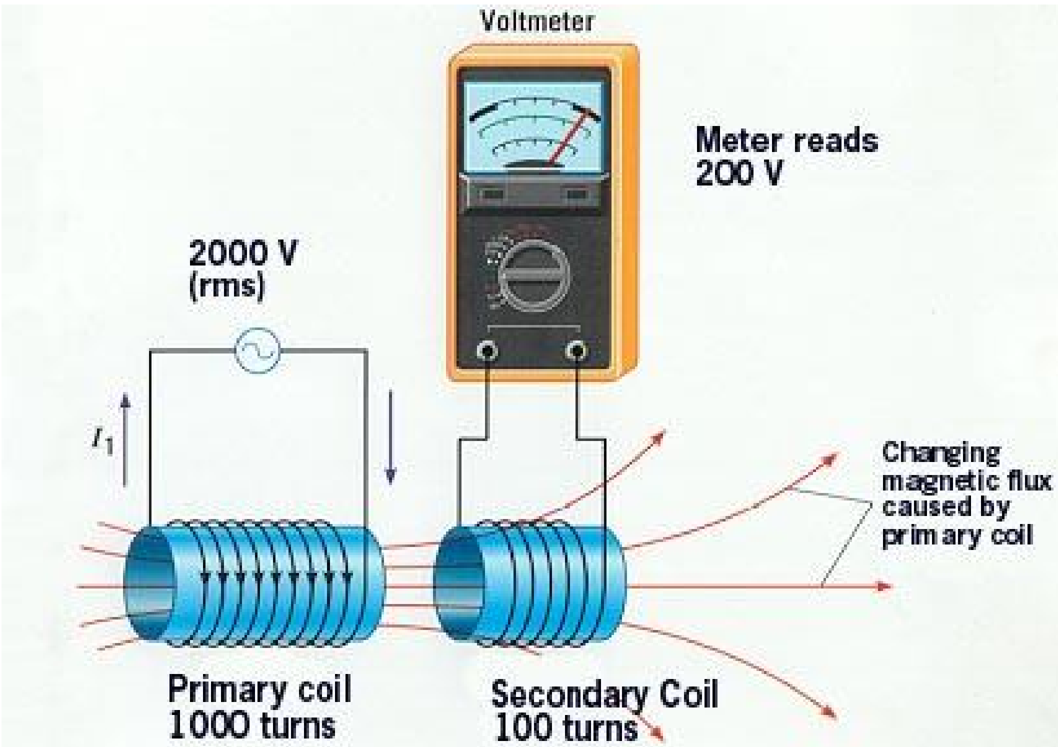
Transformer

An A.C. device used to change high voltage low current A.C. into low voltage high current A.C. and vice-versa without changing the frequency

In brief,

1. Transfers electric power from one circuit to another
2. It does so without a change of frequency
3. It accomplishes this by electromagnetic induction
4. Where the two electric circuits are in mutual inductive influence of each other.

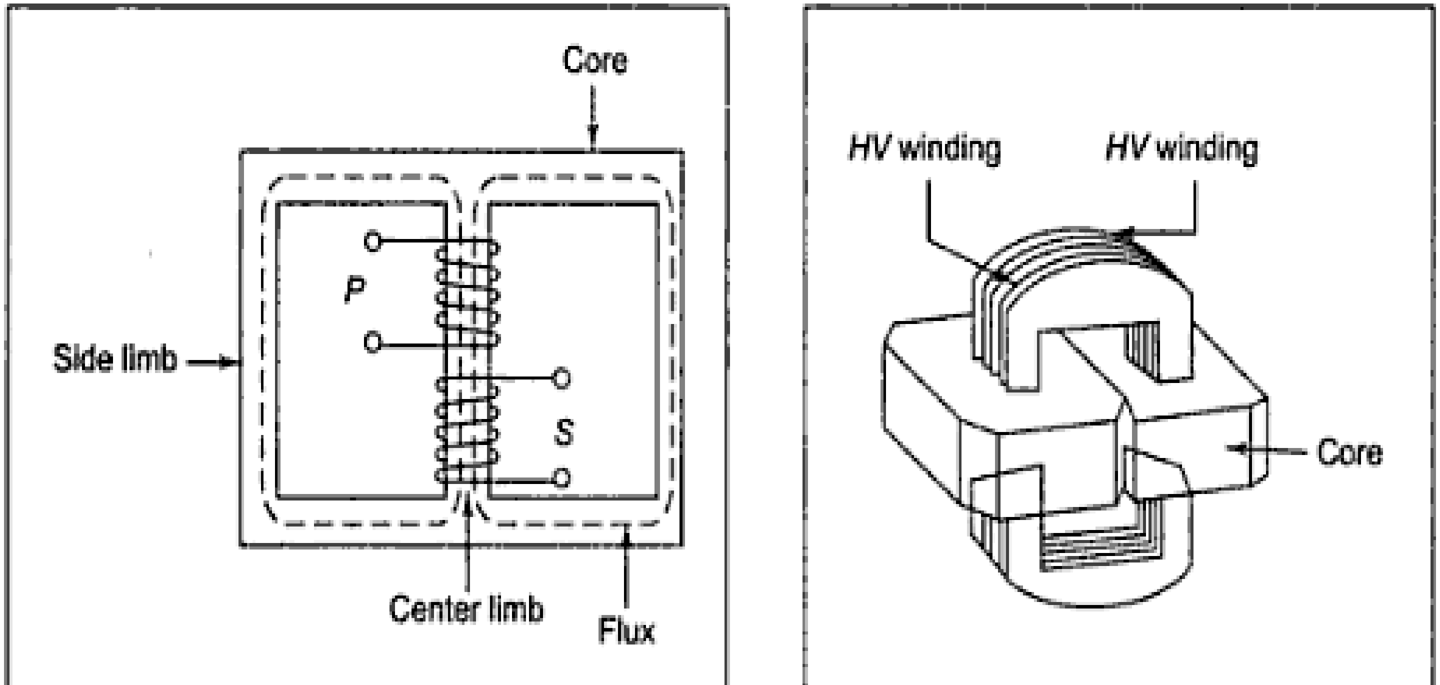
Principle of operation



It is based on principle of MUTUAL INDUCTION.

According to which an e.m.f. is induced in a coil when current in the neighbouring coil changes.

Constructional detail : Shell type

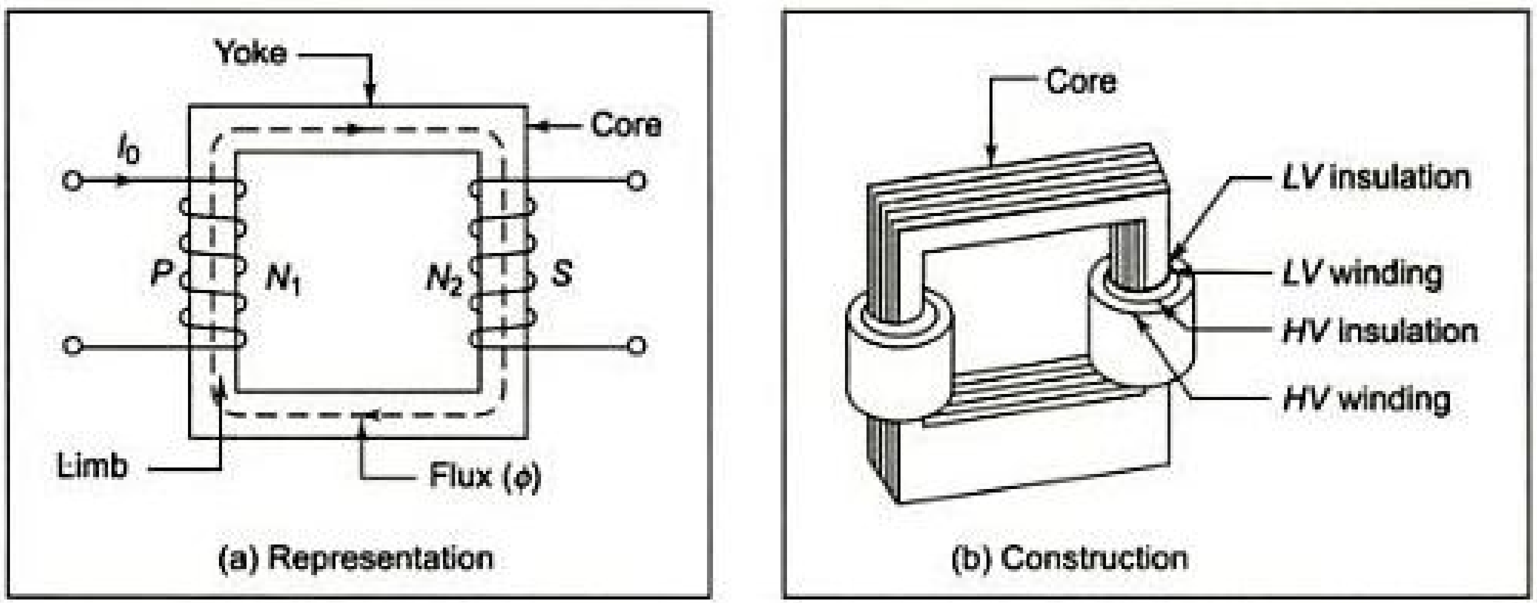


(a) Representation

(b) Construction

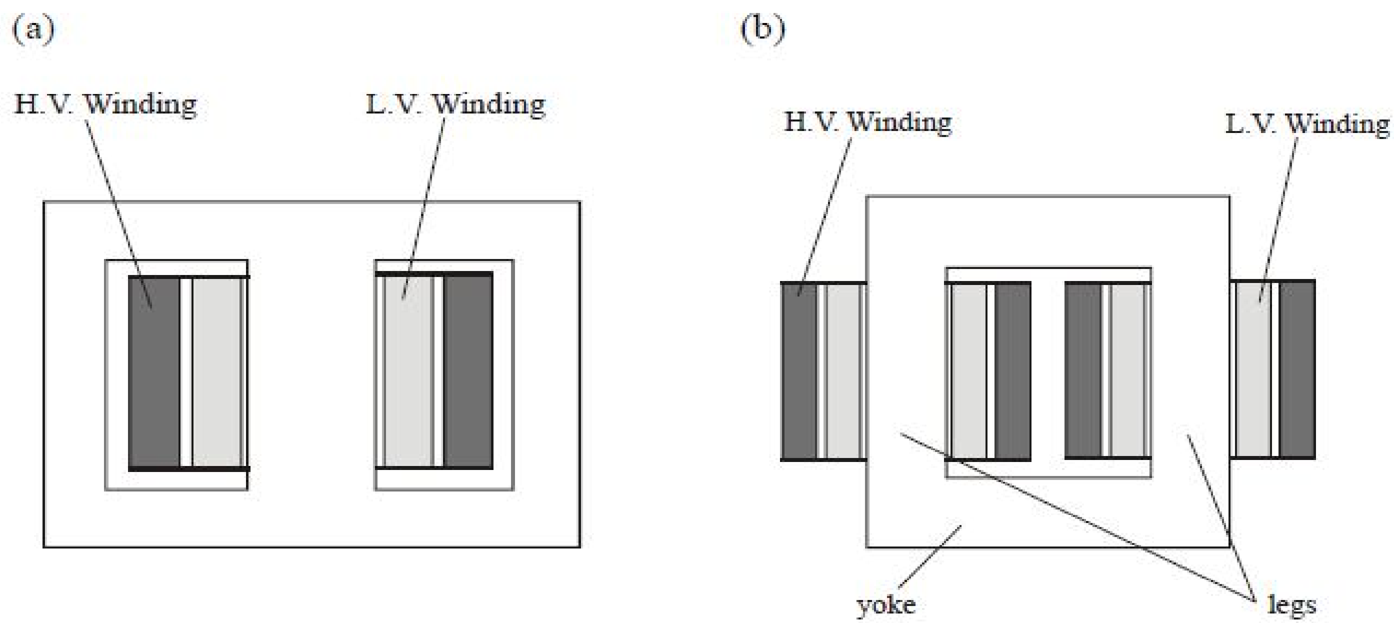
- Windings are wrapped around the center leg of a laminated core.

Core type



- Windings are wrapped around two sides of a laminated square core.

Sectional view of transformers

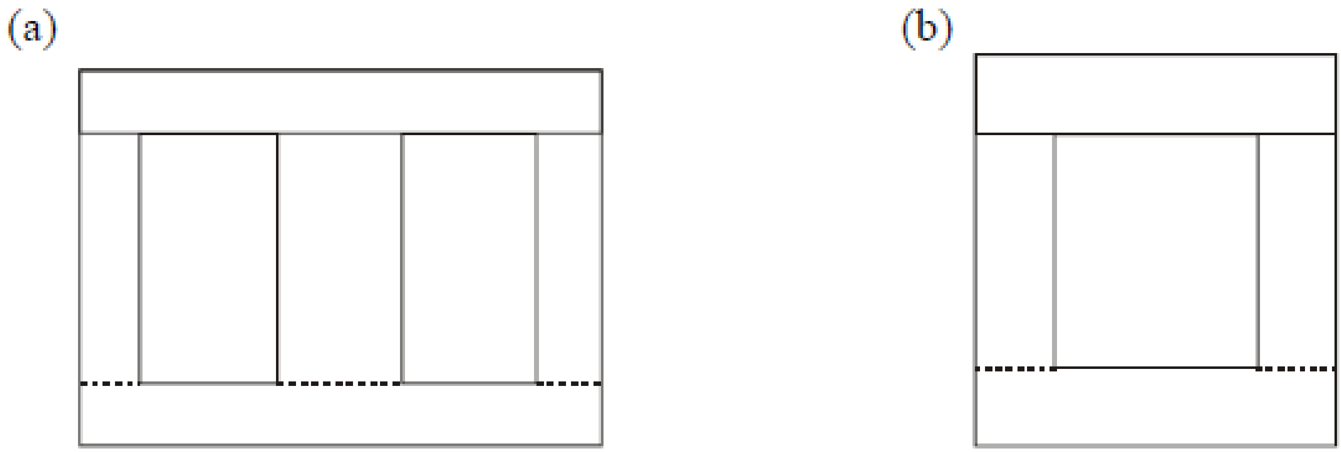


(a) Shell-type transformer, (b) core-type transformer

Note:

High voltage conductors are smaller cross section conductors than the low voltage coils

Construction of transformer from stampings



(a) Shell-type transformer, (b) core-type transformer

Core type

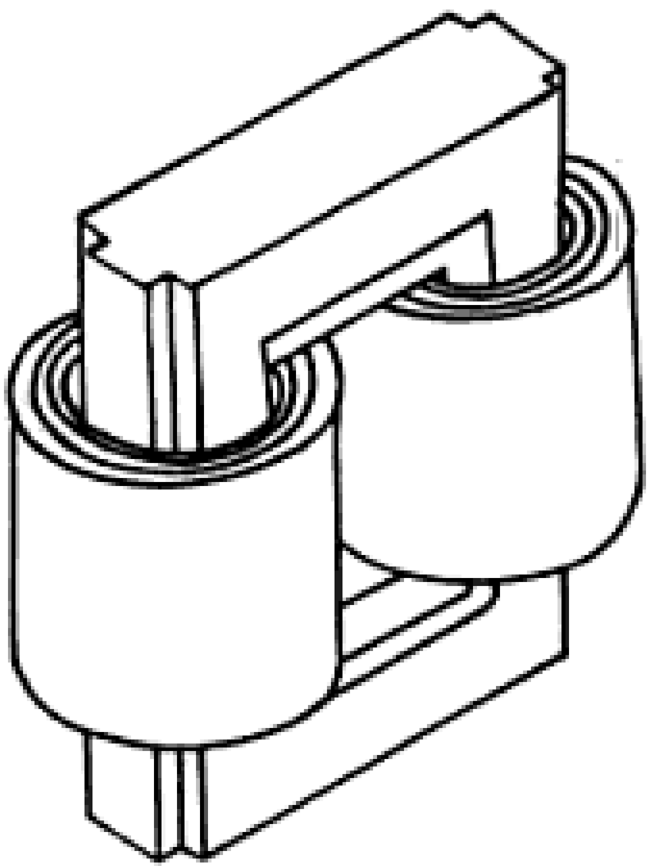


Fig1: Coil and laminations of core type transformer

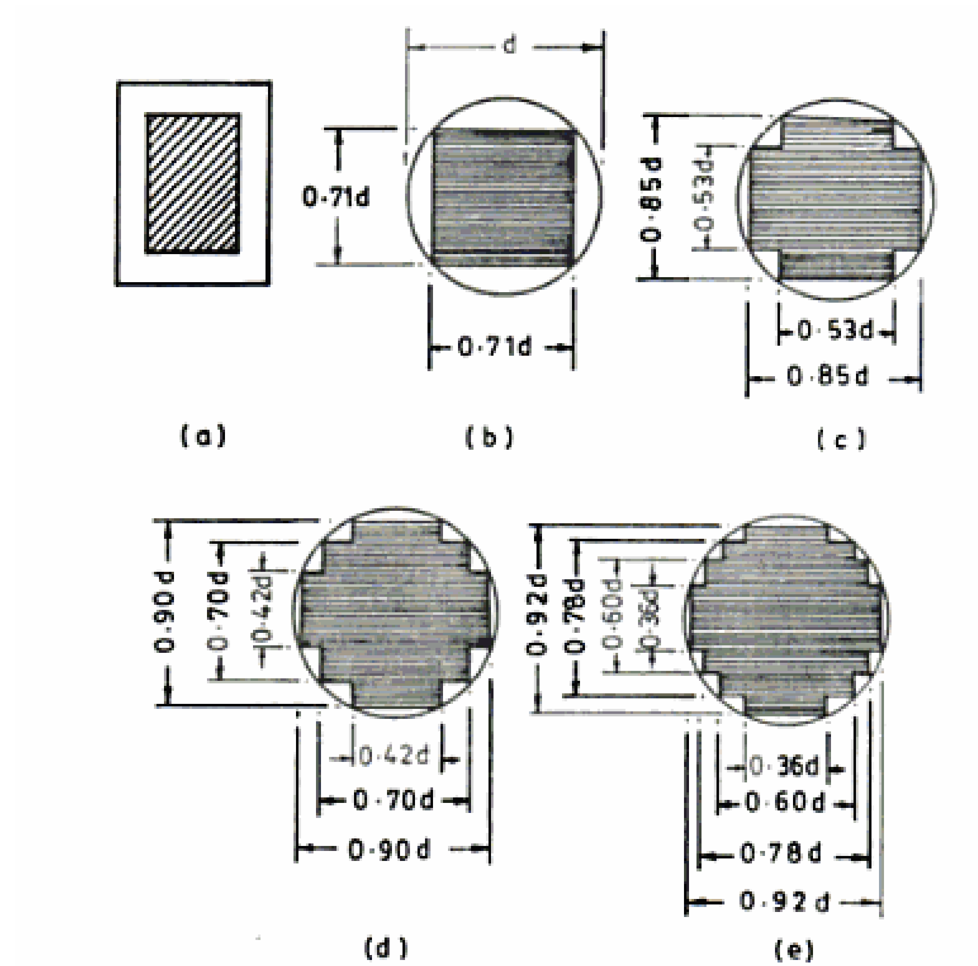


Fig2: Various types of cores

Shell type

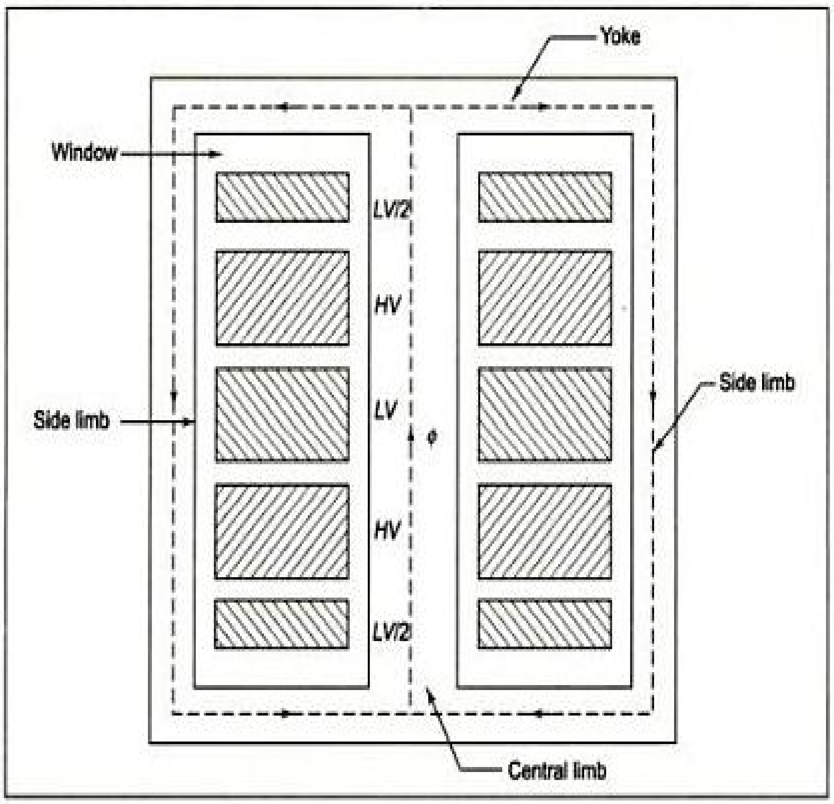
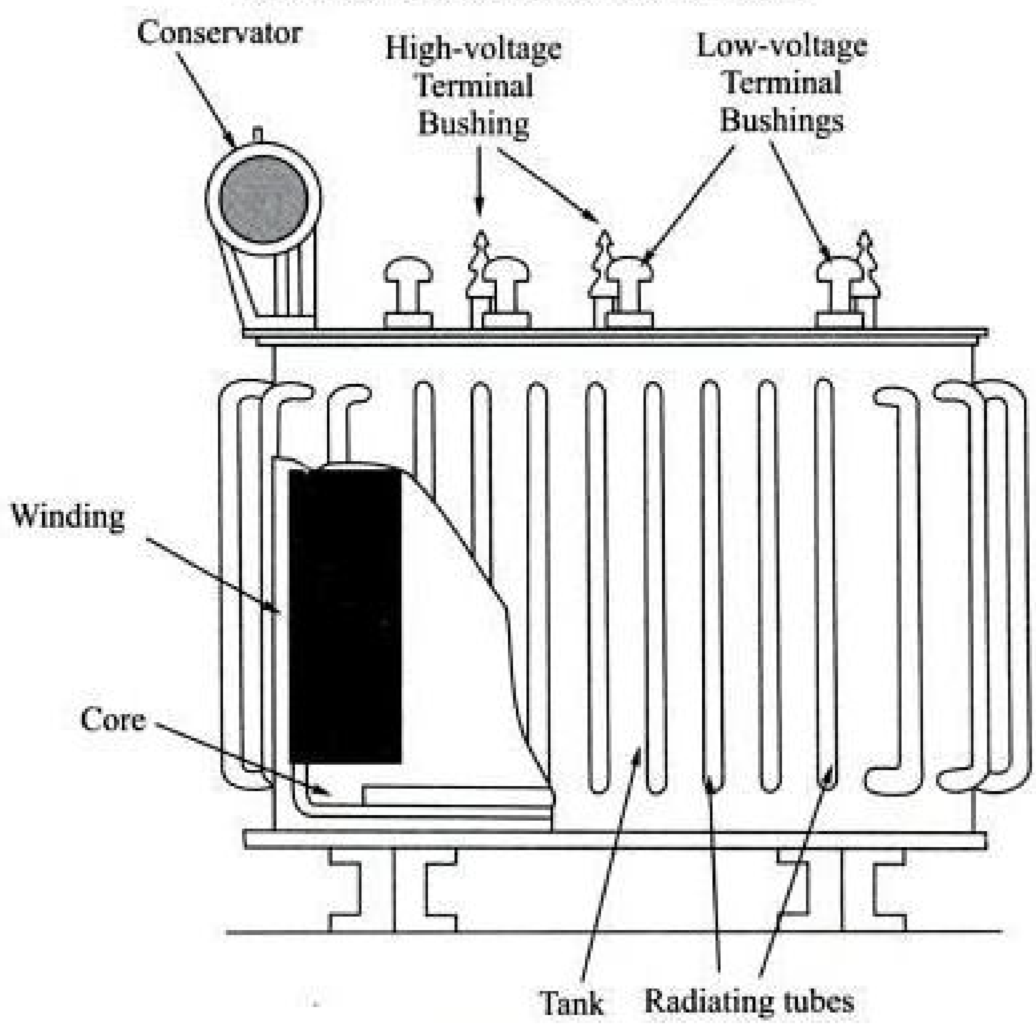


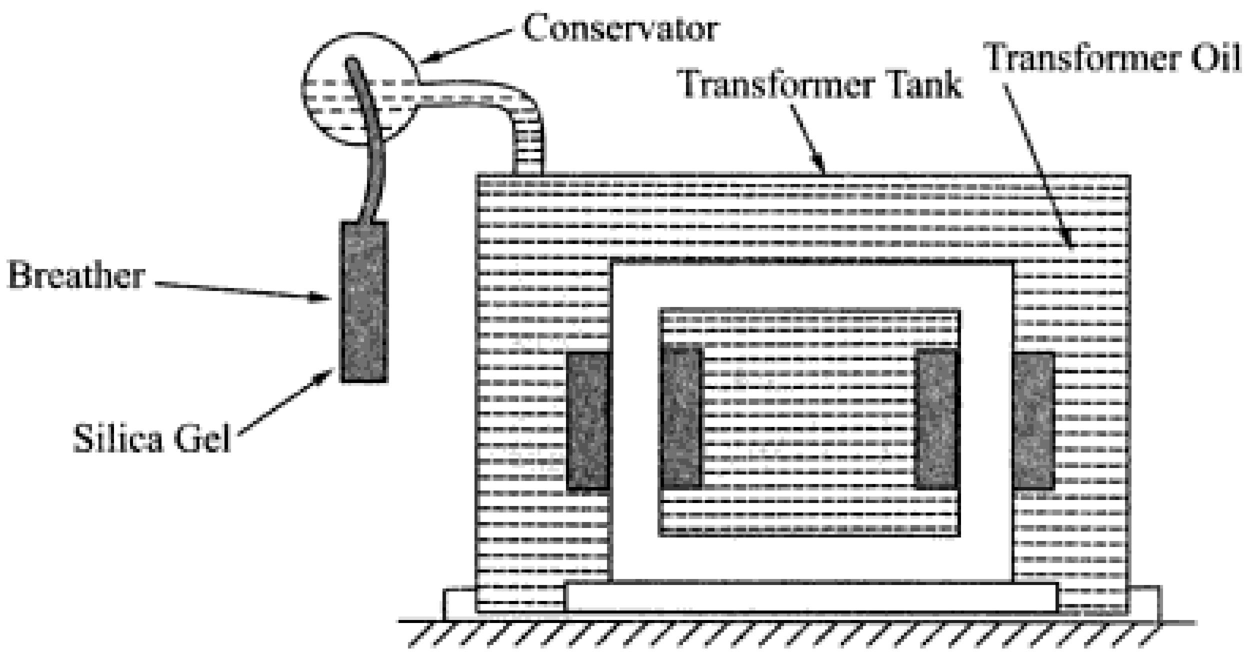
Fig: Sandwich windings

- The HV and LV windings are split into no. of sections
- Where HV winding lies between two LV windings
- In sandwich coils leakage can be controlled

Cut view of transformer

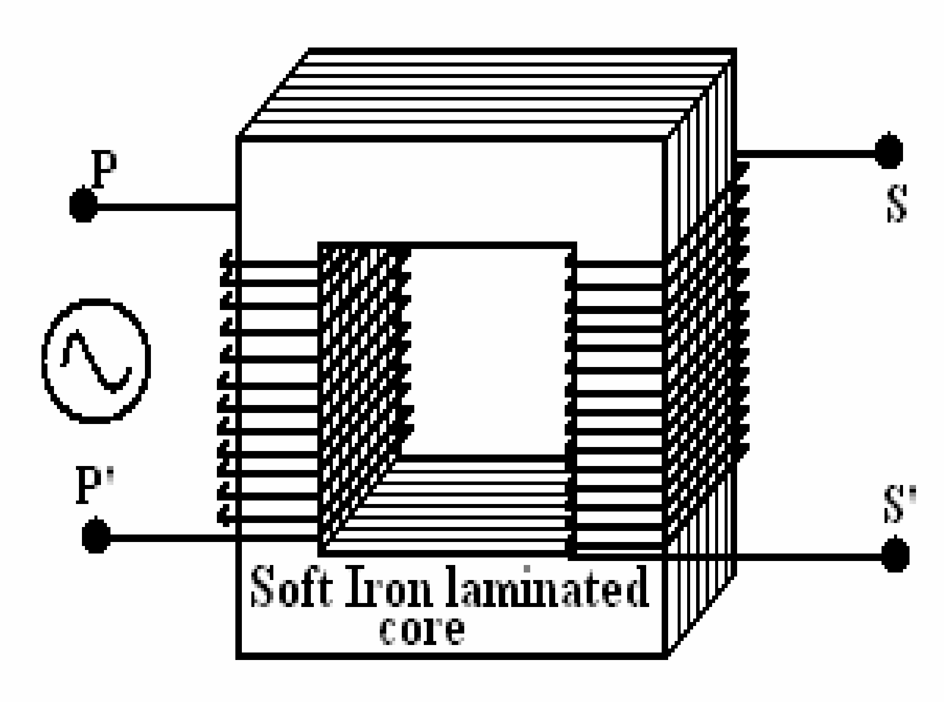


Transformer with conservator and breather



Working of a transformer

1. When current in the primary coil changes being alternating in nature, a changing magnetic field is produced
2. This changing magnetic field gets associated with the secondary through the soft iron core
3. Hence magnetic flux linked with the secondary coil changes.
4. Which induces e.m.f. in the secondary.



Single Phase Transformer

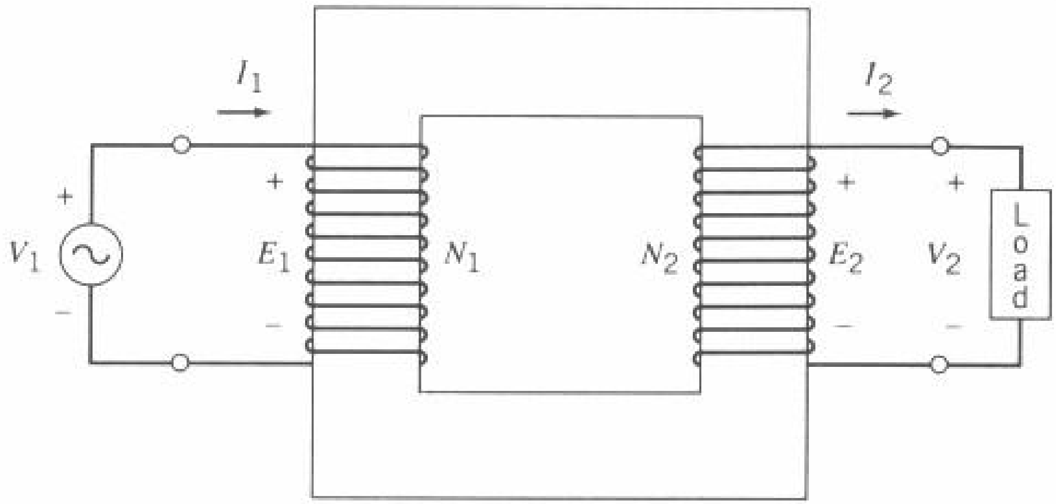


FIGURE 4.8 A transformer circuit.

- A single phase transformer
  - Two or more winding, coupled by a common magnetic core

Ideal Transformers

- **Zero leakage flux:**

  -Fluxes produced by the primary and secondary currents are confined within the core

- **The windings have no resistance:**

  - Induced voltages equal applied voltages

- **The core has infinite permeability**

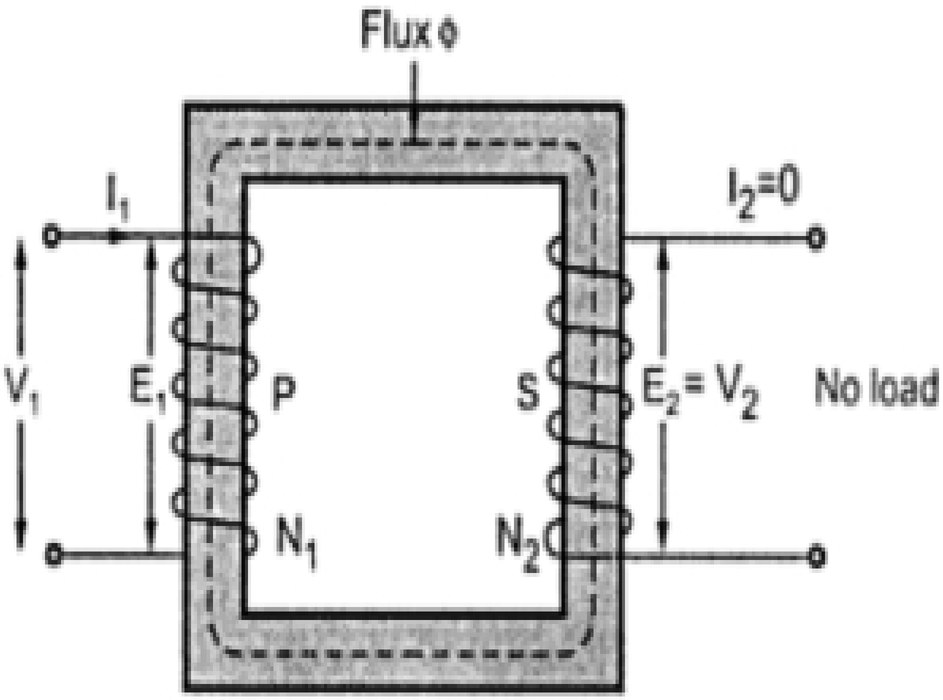
  - Reluctance of the core is zero

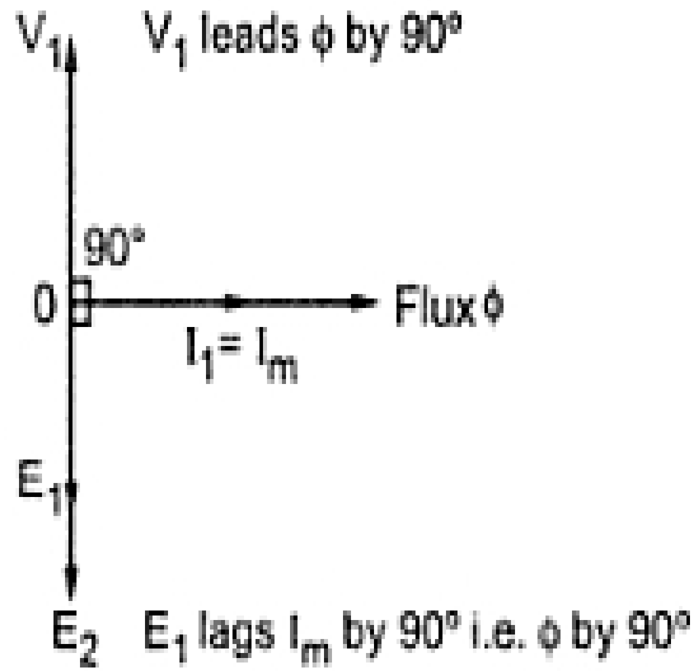
  - Negligible current is required to establish magnetic flux

- **Loss-less magnetic core**

  - No hysteresis or eddy currents

Ideal transformer





V_1 – supply voltage ; I_1- noload input current ;

V_{2-} output voltgae; I_2- output current

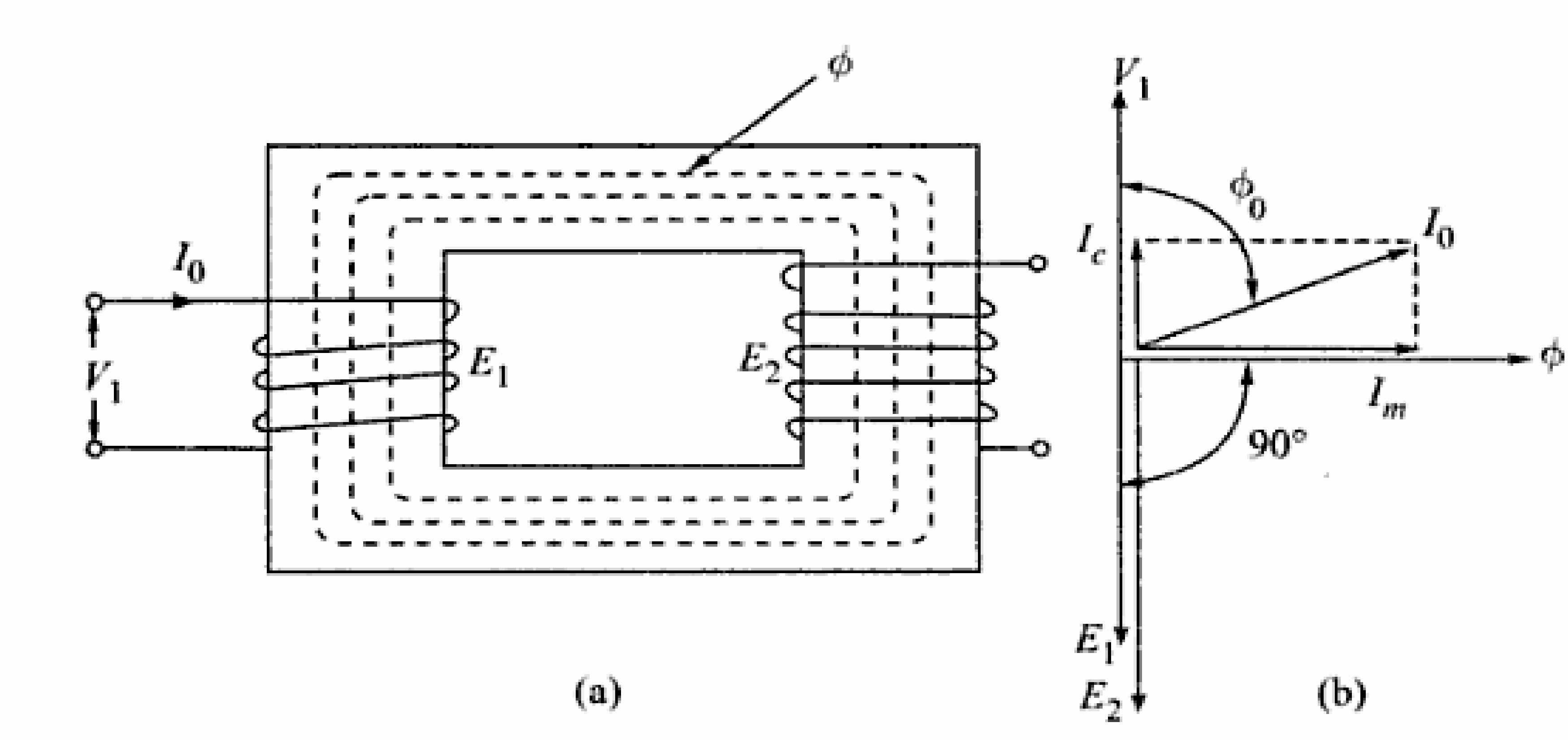
I_m- magnetising current;

E_1-self induced emf ; E_2- mutually induced emf

EMF equation of a transformer

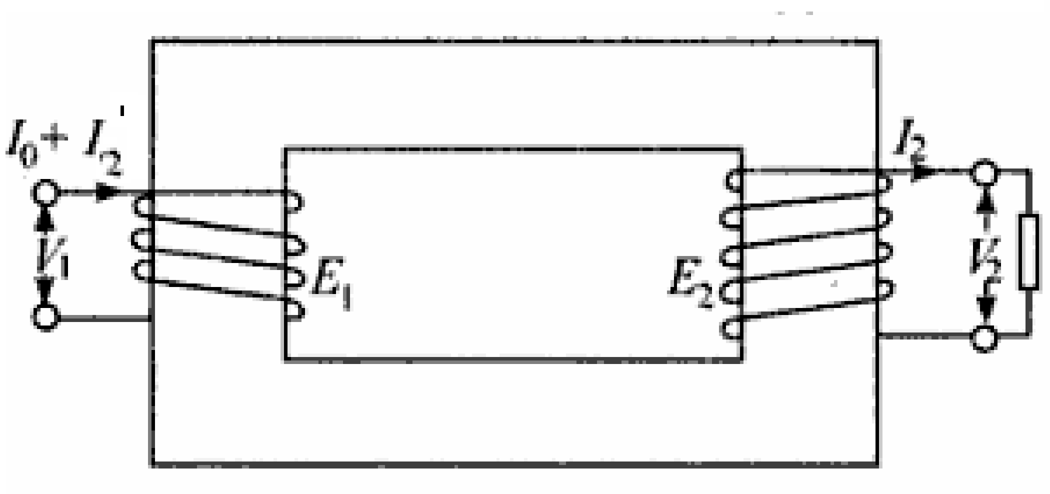
- Worked out on board /
- Refer pdf file: emf-equation-of-tranformer

Phasor diagram: Transformer on No-load



(a) Transformer on no-load (b) Phasor diagram of a transformer on no-load

Transformer on load assuming no voltage drop in the winding



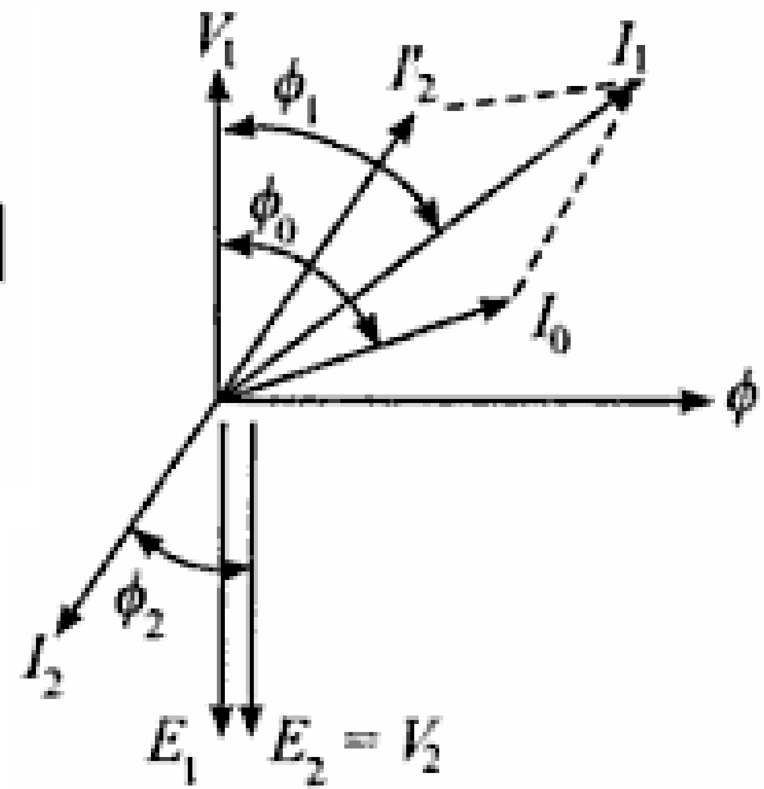


Fig shows the Phasor diagram of a transformer on load by assuming

1. No voltage drop in the winding
2. Equal no. of primary and secondary turns

Transformer on load

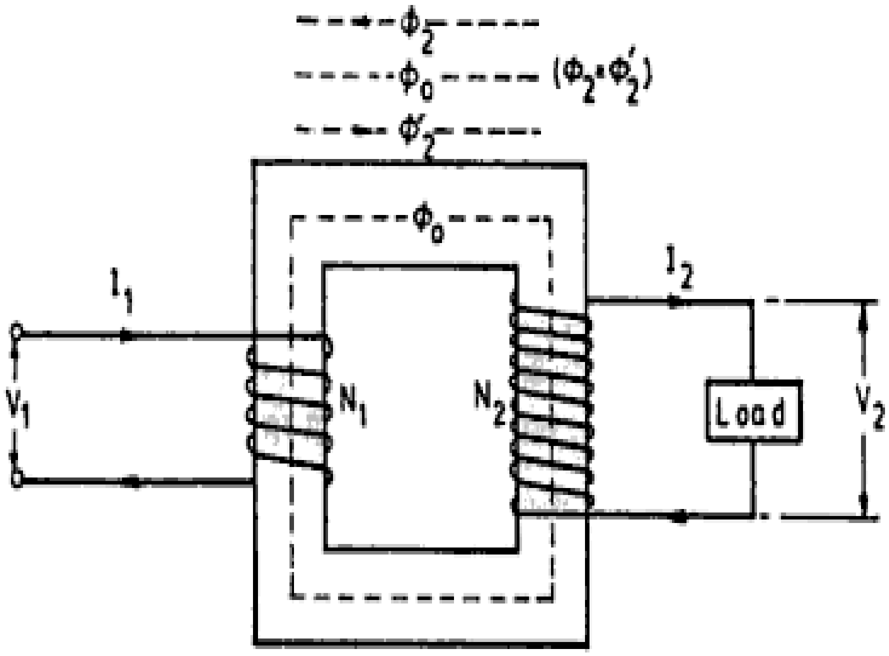


Fig. a: Ideal transformer on load

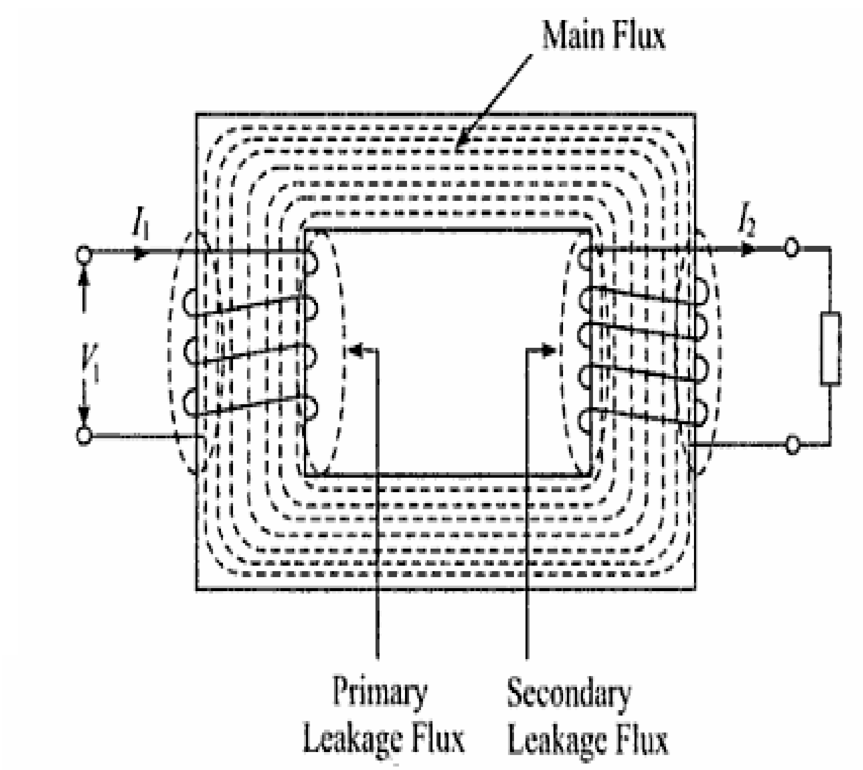
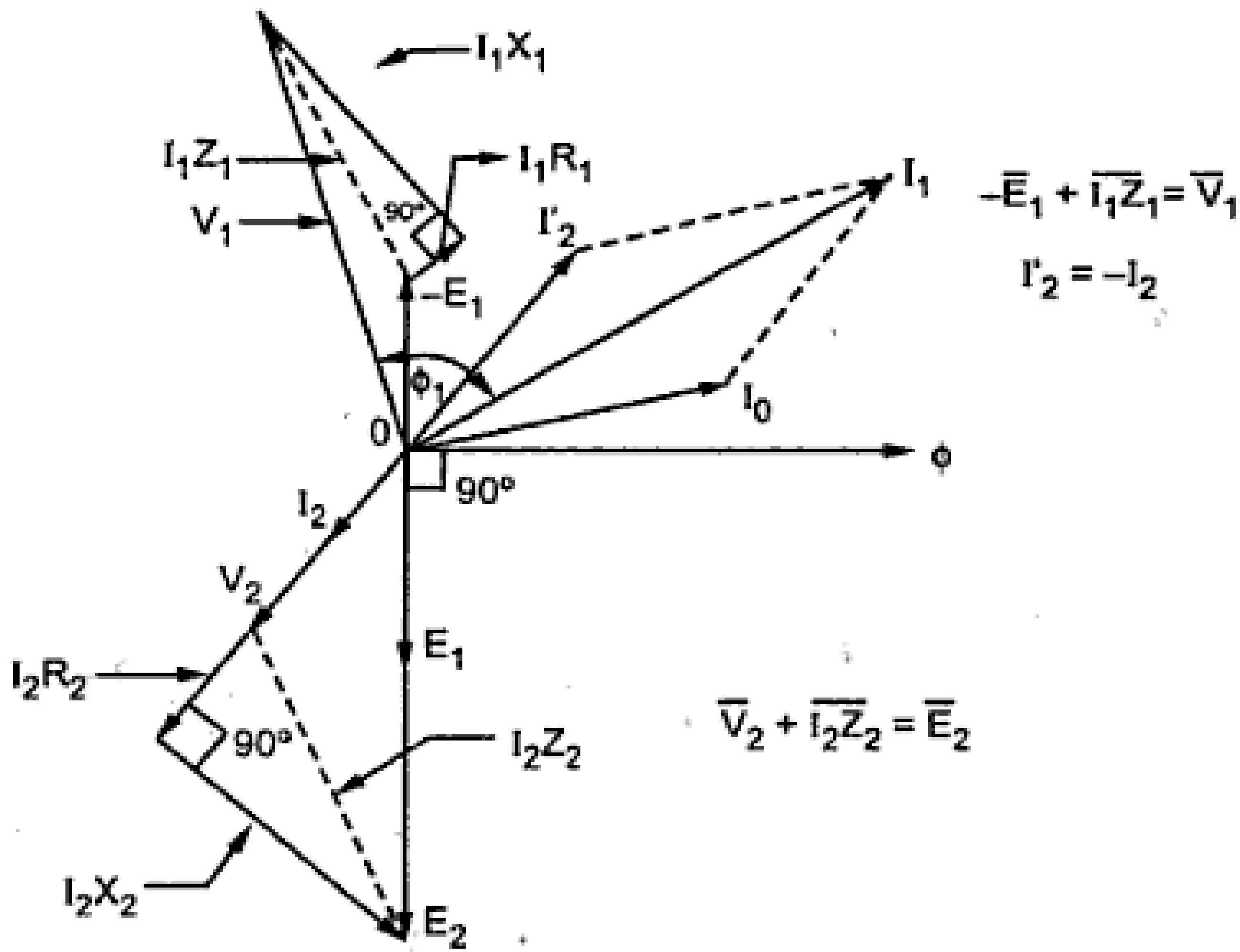
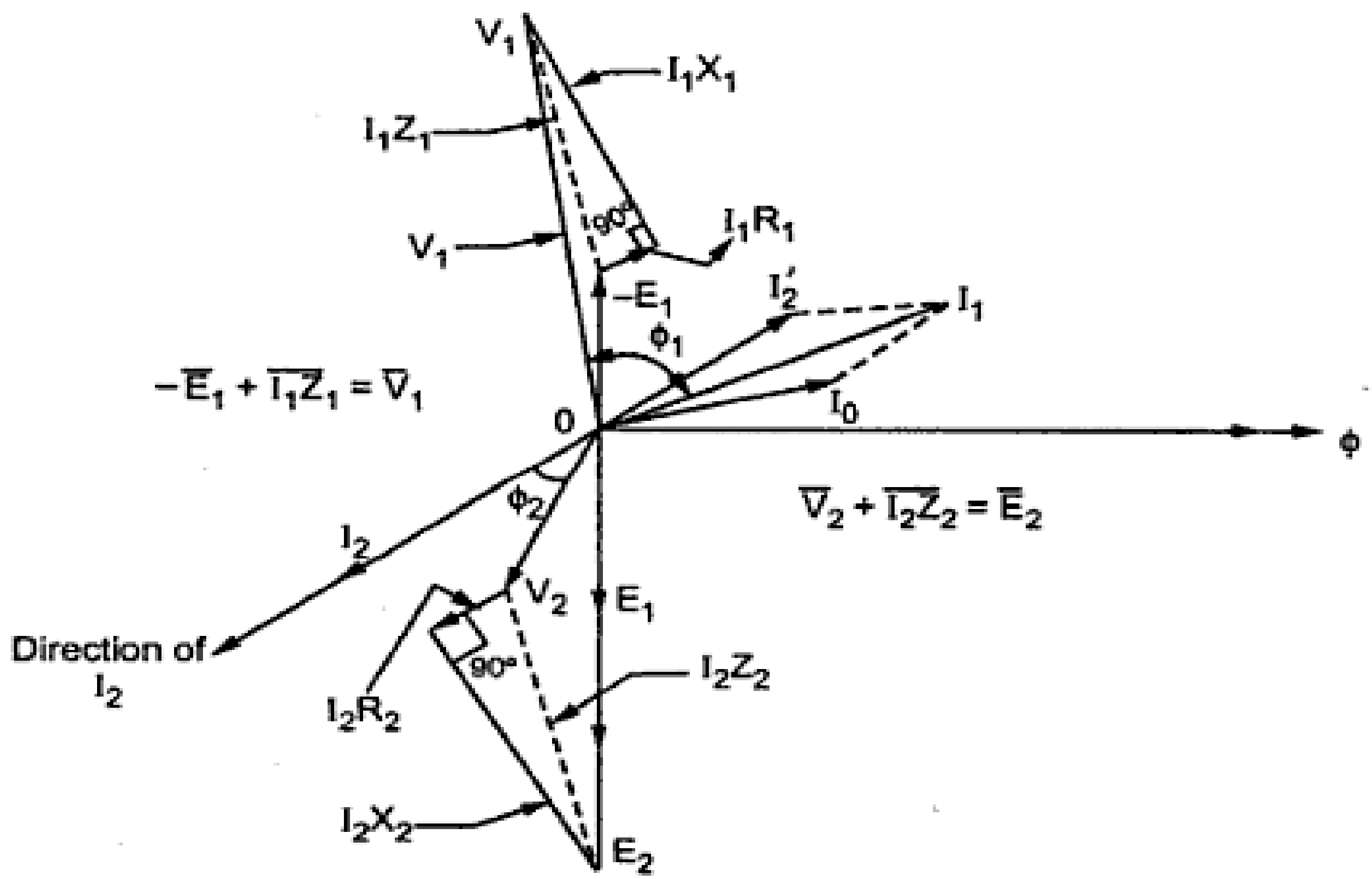


Fig. b: Main flux and leakage flux in a transformer

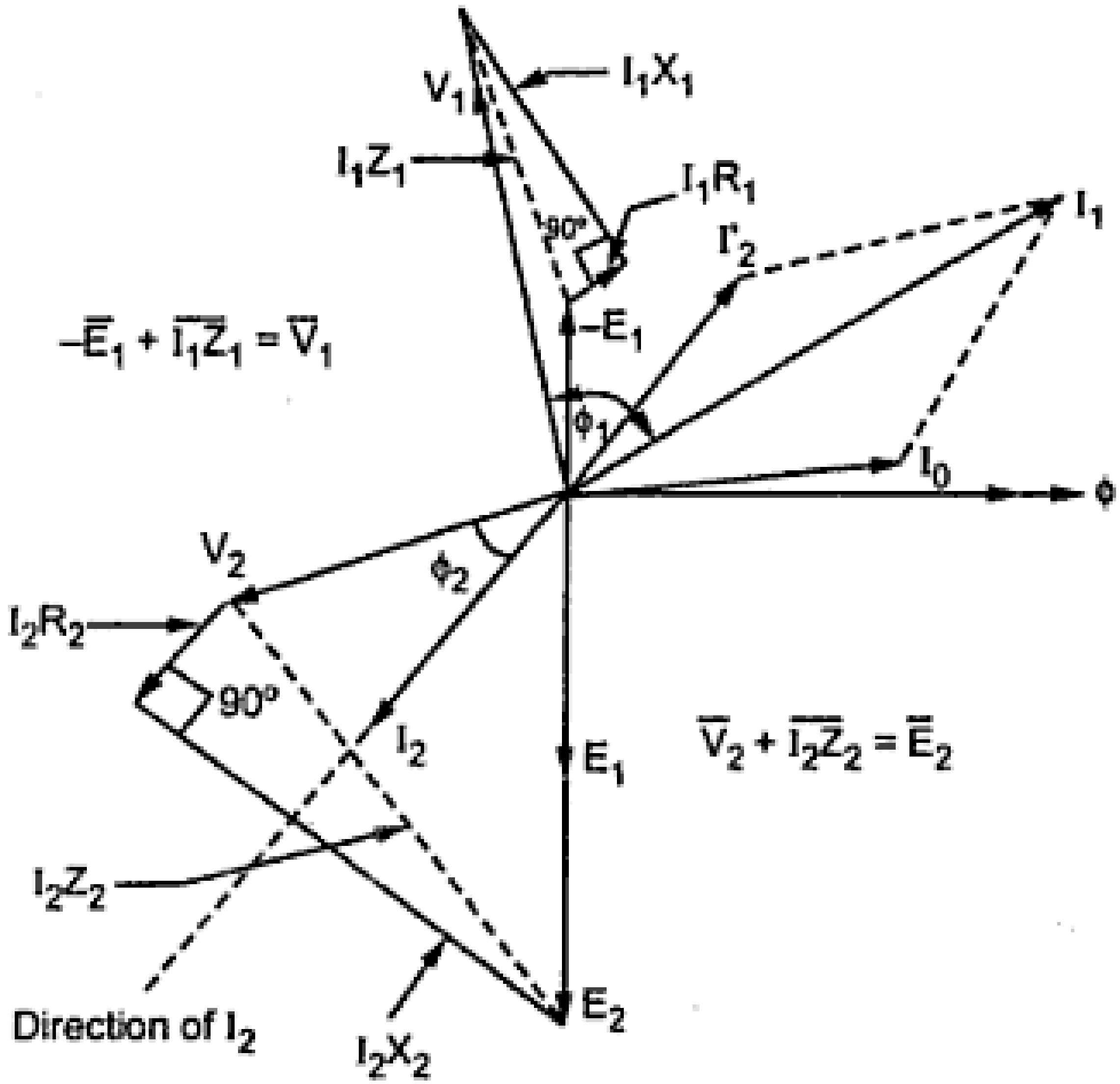
Phasor diagram of transformer with UPF load



Phasor diagram of transformer with lagging p.f load

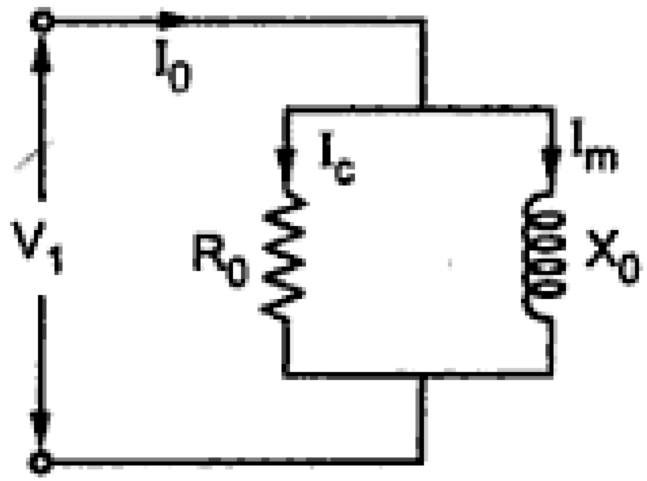


Phasor diagram of transformer with leading p.f load



Equivalent circuit of a transformer

No load equivalent circuit:



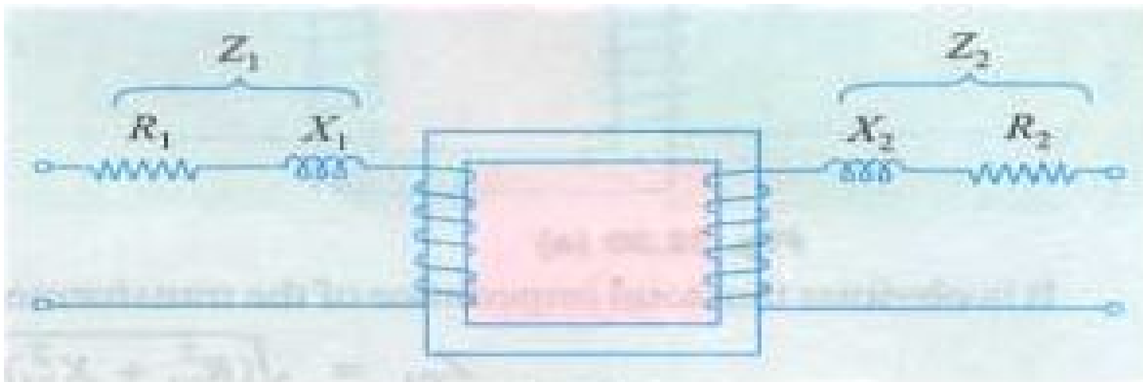
$$R_0 = \frac{V_1}{I_c}$$

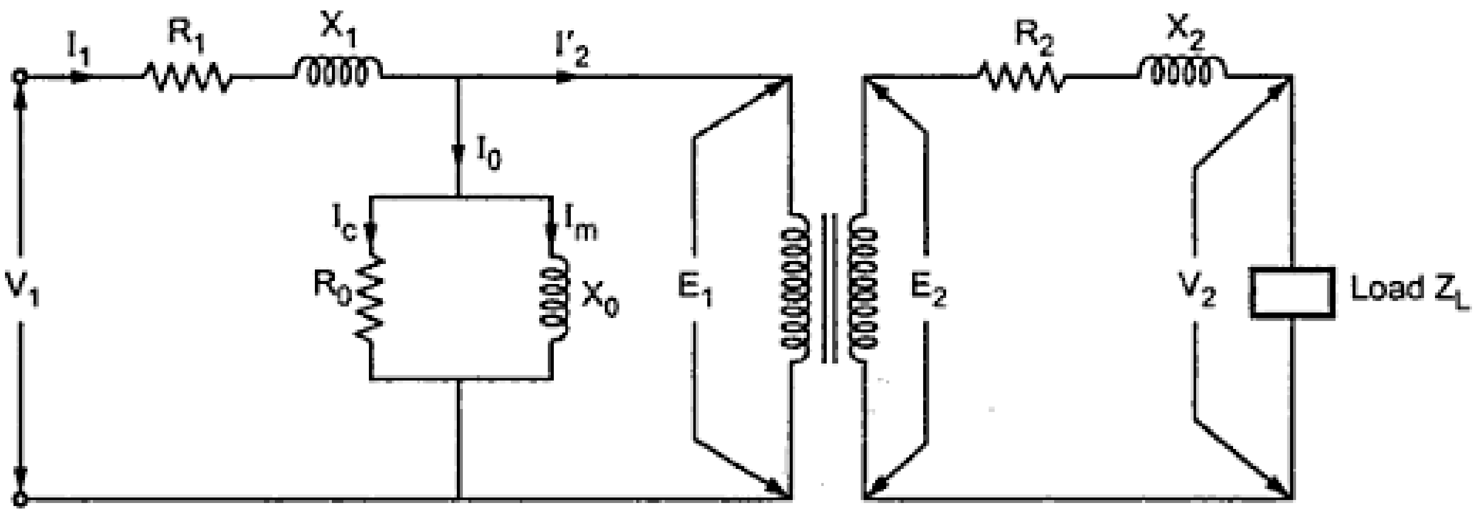
$$X_0 = \frac{V_1}{I_m}$$

$$I_m = I_0 \sin \phi_0 = \text{Magnetising component}$$

$$I_c = I_0 \cos \phi_0 = \text{Active component}$$

Equivalent circuit parameters referred to primary and secondary sides respectively





Contd.,

- The effect of circuit parameters shouldn't be changed while transferring the parameters from one side to another side
- It can be proved that a resistance of R_2 in sec. is equivalent to R_2/k^2 will be denoted as R_2'(ie. Equivalent sec. resistance w.r.t primary) which would have caused the same loss as R_2 in secondary,

$$I_1^2 R_2^{'} = I_2^2 R_2$$

$$\mathrm{R}_2^{'} = \left(\frac{I_2}{I_1}\right)^2 R_2$$

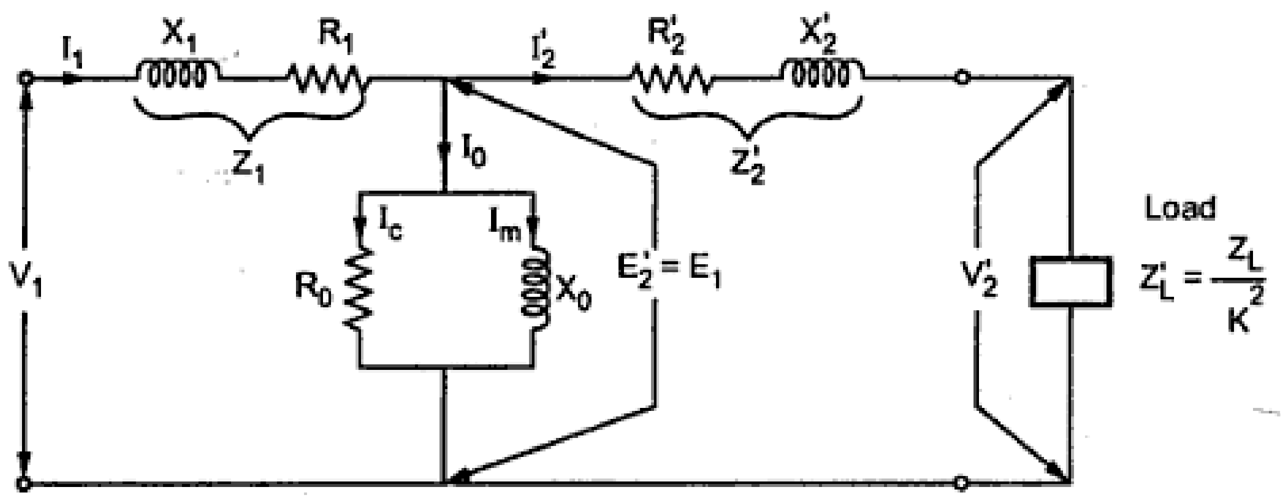
$$= \frac{\mathrm{R}_2}{\mathrm{k}^2}$$

Transferring secondary parameters to primary side

$$R_2' = \frac{R_2}{K^2}, \qquad X_2' = \frac{X_2}{K^2}, \qquad Z_2' = \frac{Z_2}{K^2}$$

While $$E_2' = \frac{E_2}{K}, \qquad I_2' = KI_2$$

where $$K = \frac{N_2}{N_1}$$



Exact equivalent circuit referred to primary

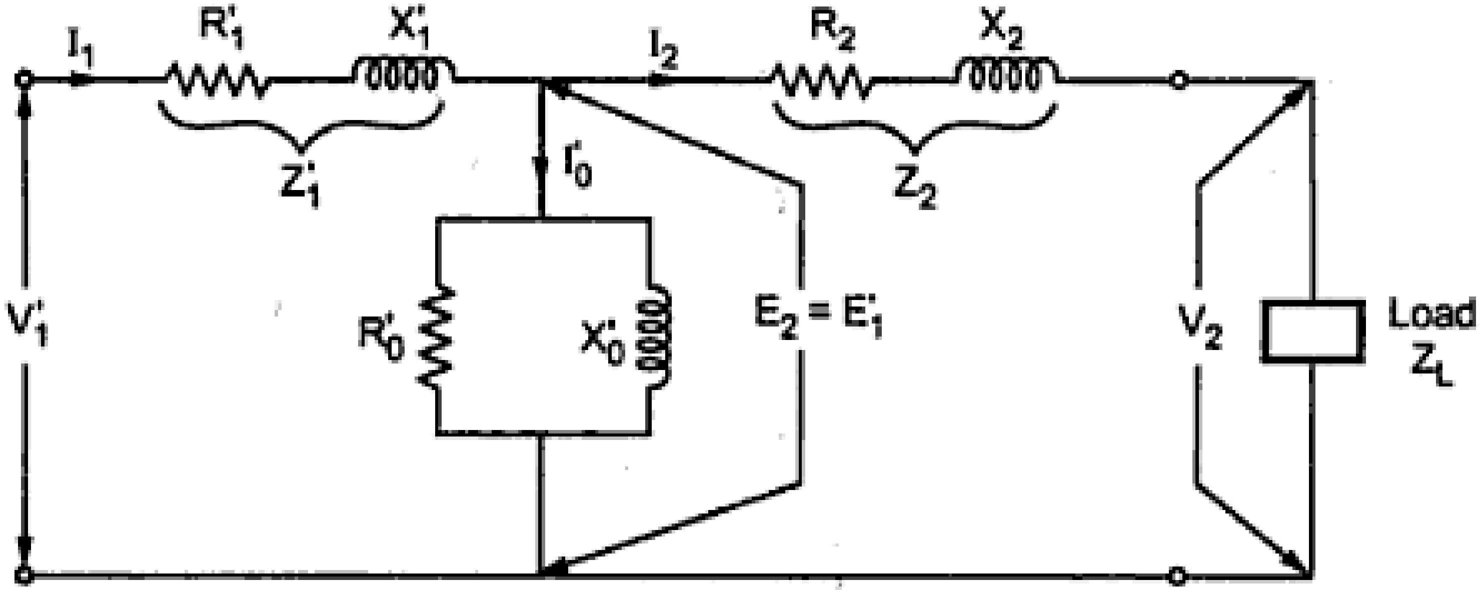
Equivalent circuit referred to secondary side

- Transferring primary side parameters to secondary side

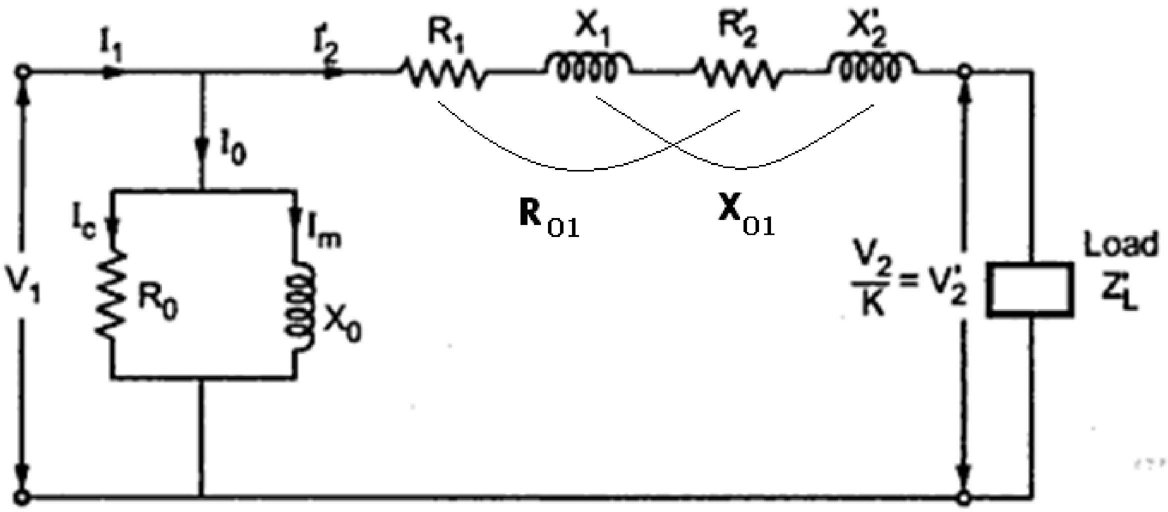
$$R_1' = K^2 R_1, \quad X_1' = K^2 X_1, \quad Z_1' = K^2 Z_1$$

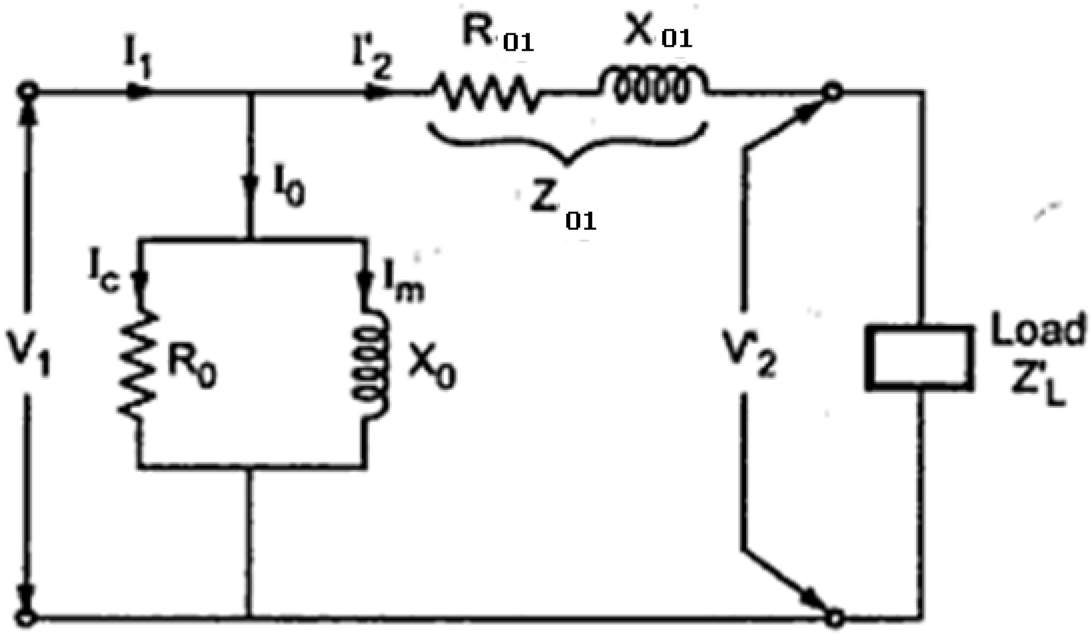
$$E_1' = K E_1, \quad I_1' = \frac{I_1}{K}, \quad I_0' = \frac{I_0}{K}$$

Similarly exciting circuit parameters are also transferred to secondary as R_o' and X_o'



equivalent circuit w.r.t primary





where

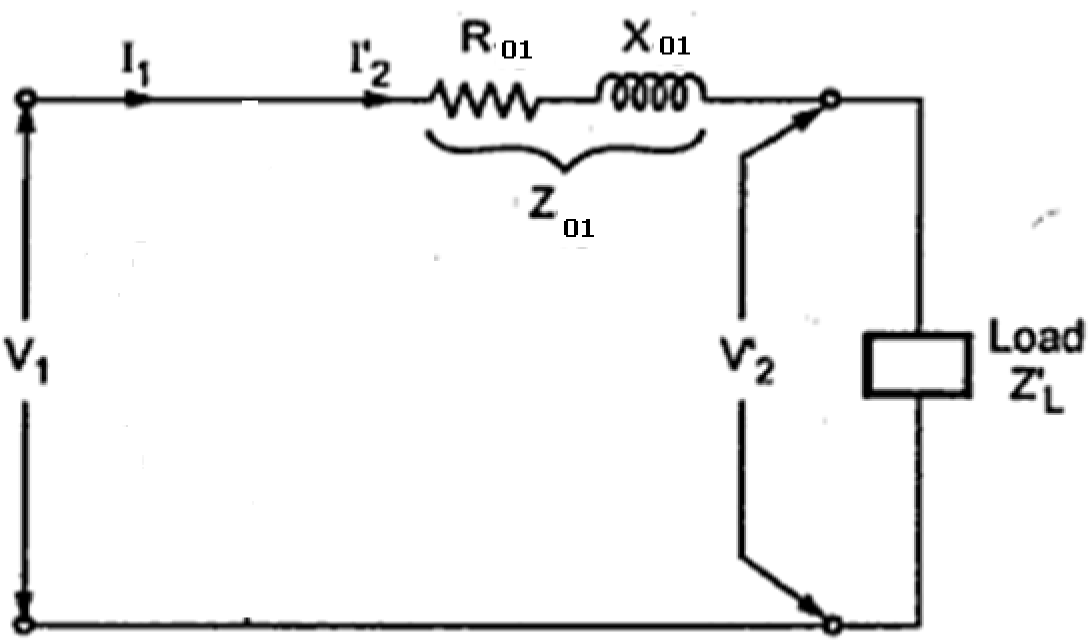
$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$

$$Z_{01} = R_{01} + j\,X_{01}$$

Approximate equivalent circuit

- Since the noload current is 1% of the full load current, the nolad circuit can be neglected

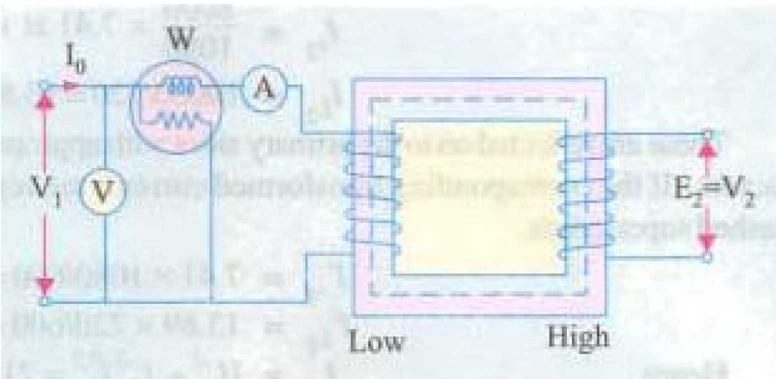


Transformer Tests

- The performance of a transformer can be calculated on the basis of equivalent circuit
- The four main parameters of equivalent circuit are:
  - R_{01} as referred to primary (or secondary R_{02})
  - the equivalent leakage reactance X_{01} as referred to primary (or secondary X_{02})
  - Magnetising susceptance B_0 (or reactance X_0)
  - core loss conductance G_0 (or resistance R_0)
- The above constants can be easily determined by two tests
  - Oper circuit test (O.C test / No load test)
  - Short circuit test (S.C test/Impedance test)
- These tests are economical and convenient
  - these tests furnish the result without actually loading the transformer

Open-circuit Test

In Open Circuit Test the transformer's ***secondary winding is open-circuited***, and its ***primary winding is connected to a full-rated line voltage***.



$$\text{Core loss} = W_{oc} = V_0 I_0 \cos\phi_0$$

$$\cos\phi_0 = \frac{W_{oc}}{V_0 I_0}$$

$$I_c \, or \, I_w = I_0 \cos\phi_0$$

$$I_m \, or \, I_\mu = I_0 \sin\phi_0 = \sqrt{I_0^2 - I_w^2}$$

$$I_0 = V_0 Y_0; \quad \therefore Y_o = \frac{I_0}{V_0}$$

$$R_0 = \frac{V_0}{I_w}$$

$$X_0 = \frac{V_0}{I\mu}$$

$$G_0 = \frac{I_w}{V_0}$$

$$B_0 = \frac{I\mu}{V_0}$$

$$W_{oc} = V_0^2 G_0; \quad \therefore \text{Exciting conductance } G_0 = \frac{W_{oc}}{V_0^2}$$

$$\& \quad \text{Exciting susceptance } B_0 = \sqrt{Y_0^2 - G_0^2}$$

- Usually conducted on H.V side
- To find

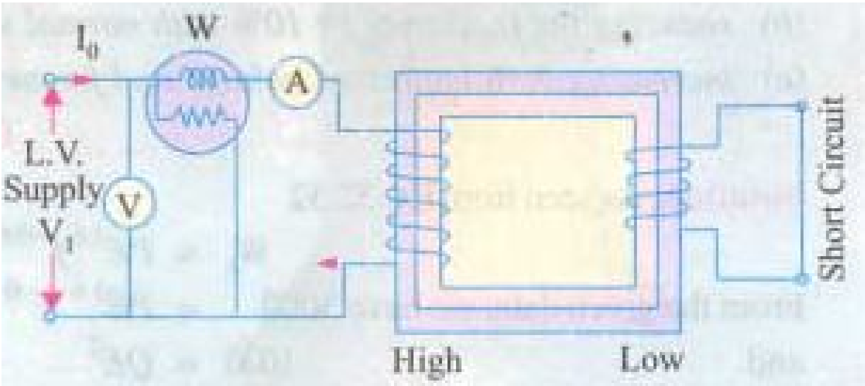
  (i) No load loss or core loss

  (ii) No load current I_o which is helpful in finding G_o(or R_o) and B_o (or X_o)

Short-circuit Test

In Short Circuit Test the ***secondary terminals are short circuited***, and the ***primary terminals are connected to a fairly low-voltage source***

The input voltage is adjusted until the current in the short circuited windings is equal to its rated value. The input voltage, current and power is measured.



- Usually conducted on L.V side
- To find

  (i) Full load copper loss – to pre determine the efficiency

  (ii) Z_{01} or Z_{02}; X_{01} or X_{02}; R_{01} or R_{02} - to predetermine the voltage regulation

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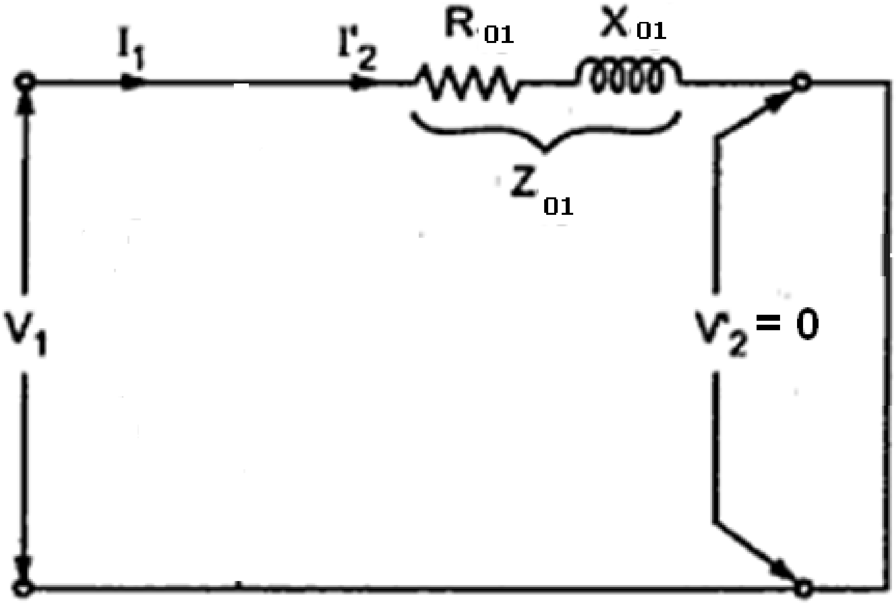


$$\text{Full load cu loss} = W_{sc} = I_{sc}^2 R_{01}$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$\therefore X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

Transformer Voltage Regulation and Efficiency

The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it. Full load Voltage Regulation is a quantity that compares the output voltage at no load with the output voltage at full load, defined by this equation:

$$\text{At noload} \quad k = \frac{V_s}{V_p}$$

$$\text{Regulation} \quad \text{up} = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\,\%$$

$$\text{Regulation} \quad \text{up} = \frac{(V_P / k) - V_{S,fl}}{V_{S,fl}} \, x\, 100\,\%$$

$$\text{Regulation} \quad \text{down} = \frac{V_{S,nl} - V_{S,fl}}{V_{S,nl}} \times 100\,\%$$

$$\text{Regulation} \quad \text{down} = \frac{(V_P / k) - V_{S,fl}}{V_{S,nl}} \, x\, 100\,\%$$

Ideal transformer, VR = 0%.

Voltage regulation of a transformer

$$\text{Voltage regulation} = \frac{\text{no-load voltage} - \text{full-load voltage}}{\text{no-load voltage}}$$

recall $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

Secondary voltage on no-load $V_2 = V_1\left(\frac{N_2}{N_1}\right)$

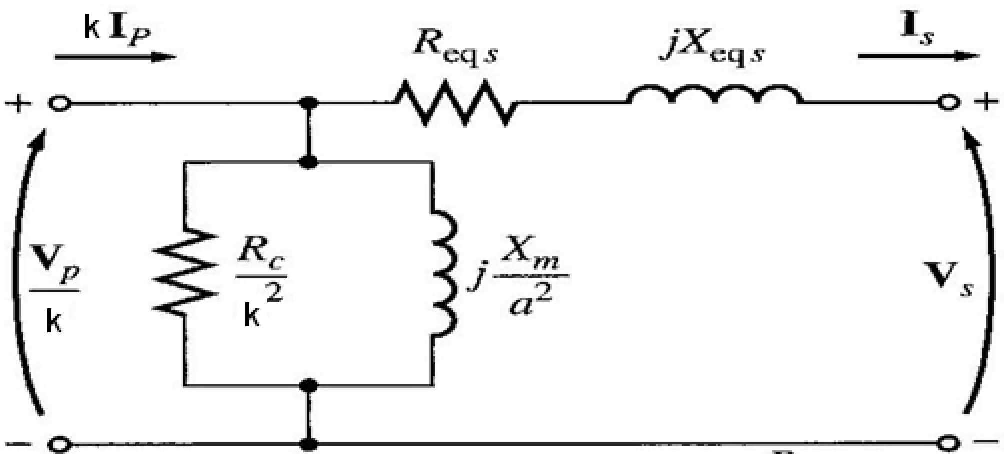
V_2 is a secondary terminal voltage on full load

Substitute we have

$$\text{Voltage regulation} = \frac{V_1\left(\frac{N_2}{N_1}\right) - V_2}{V_1\left(\frac{N_2}{N_1}\right)}$$

Transformer Phasor Diagram

To determine the voltage regulation of a transformer, it is necessary understand the voltage drops within it.



$$R_{eq\,s} = \frac{R_p}{k^2} + R_s$$

$$X_{eq\,s} = \frac{X_p}{k^2} + X_s$$

Transformer Phasor Diagram

Ignoring the excitation of the branch (since the current flow through the branch is considered to be small), more consideration is given to the series impedances (R_{eq} +jX_{eq}).

Voltage Regulation depends on magnitude of the series impedance and the phase angle of the current flowing through the transformer.

Phasor diagrams will determine the effects of these factors on the voltage regulation. A phasor diagram consist of current and voltage vectors.

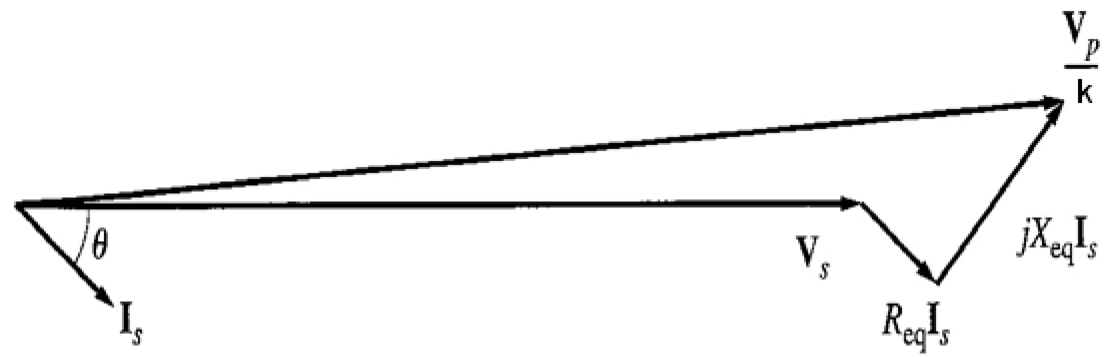
Assume that the reference phasor is the secondary voltage, V_S. Therefore the reference phasor will have 0 degrees in terms of angle.

Based upon the equivalent circuit, apply Kirchoff Voltage Law,

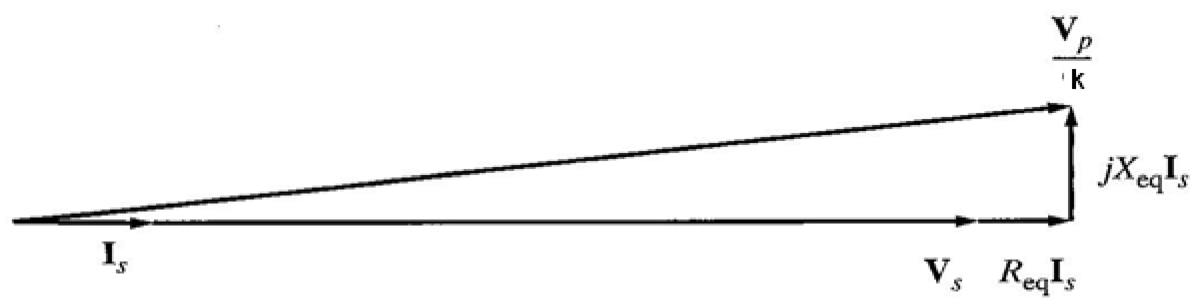
$$\frac{V_P}{k} = V_S + R_{eq} I_S + jX_{eq} I_S$$

Transformer Phasor Diagram

For lagging loads, V_P / a > V_S so the voltage regulation with lagging loads is > 0.



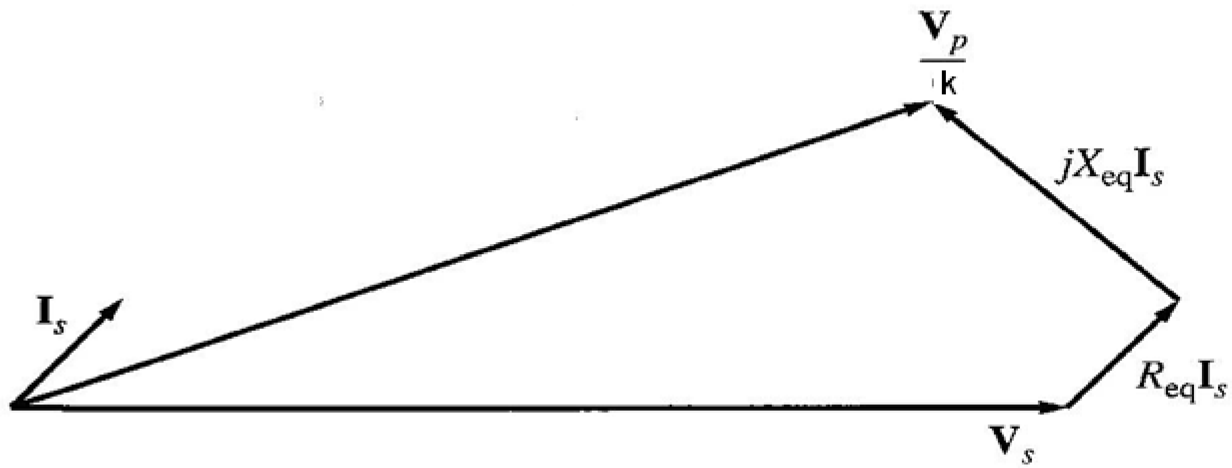
When the power factor is unity, V_S is lower than V_P so VR > 0.



(a)

Transformer Phasor Diagram

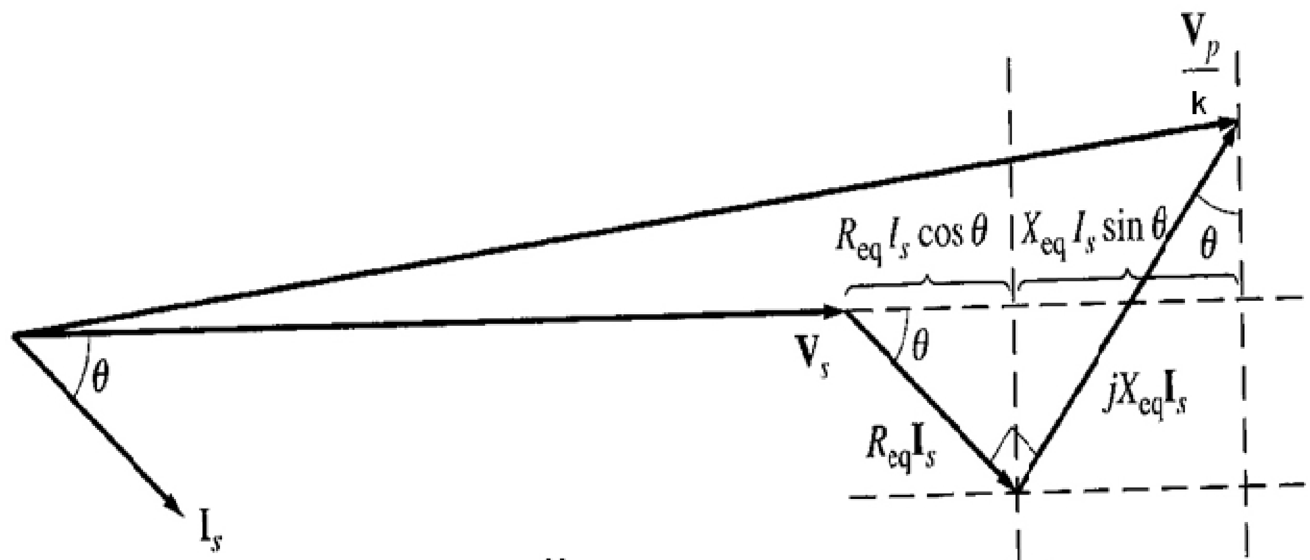
With a leading power factor, V_S is higher than the referred V_P so VR < 0



(b)

Transformer Phasor Diagram

For lagging loads, the vertical components of R_{eq} and X_{eq} will partially cancel each other. Due to that, the angle of V_P/a will be very small, hence we can assume that V_P/k is horizontal. Therefore the approximation will be as follows:



$$\frac{V_p}{k} \approx V_s + R_{eq} I_s \cos\theta + X_{eq} I_s \sin\theta$$

Formula: voltage regulation

In terms of secondary values

$$\%\ \text{regulation} = \frac{{}_0\mathrm{V}_2 - V_2}{{}_0\mathrm{V}_2} = \frac{I_2 R_{02} \cos\phi_2 \pm I_2 X_{02} \sin\phi_2}{{}_0\mathrm{V}_2}$$

where '+' for lagging and '-' for leading

In terms of primary va lues

$$\%\ \text{regulation} = \frac{\mathrm{V}_1 - V_2^{'}}{\mathrm{V}_1} = \frac{I_1 R_{01} \cos\phi_1 \pm I_1 X_{01} \sin\phi_1}{\mathrm{V}_1}$$

where '+' for lagging and '-' for leading

Transformer Efficiency

Transformer efficiency is defined as (applies to motors, generators and transformers):

$$\eta = \frac{P_{out}}{P_{in}} \times 100 \ \%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

Types of losses incurred in a transformer:

Copper I^2R losses

Hysteresis losses

Eddy current losses

Therefore, for a transformer, efficiency may be calculated using the following:

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} x100\%$$

Losses in a transformer

Core or Iron loss:

Hysteresis loss $W_h = \eta B^{1.6}{}_{max} f V$ watt;

eddy current loss $W_e = \eta B^{2}{}_{max} f^2 t^2$ watt

Copper loss:

Total Cu loss $= I_1^{\,2} R_1 + I_2^{\,2} R_2 = I_1^{\,2} R_{01} + I_2^{\,2} R_{02}$.

Contd.,

The output current corresponding to maximum efficiency is $I_2 = \sqrt{(W_i/R_{02})}$.

The load at which the two losses are equal $= \text{Full load} \times \sqrt{\left(\dfrac{\text{Iron loss}}{\text{F.L. Cu loss}}\right)}$

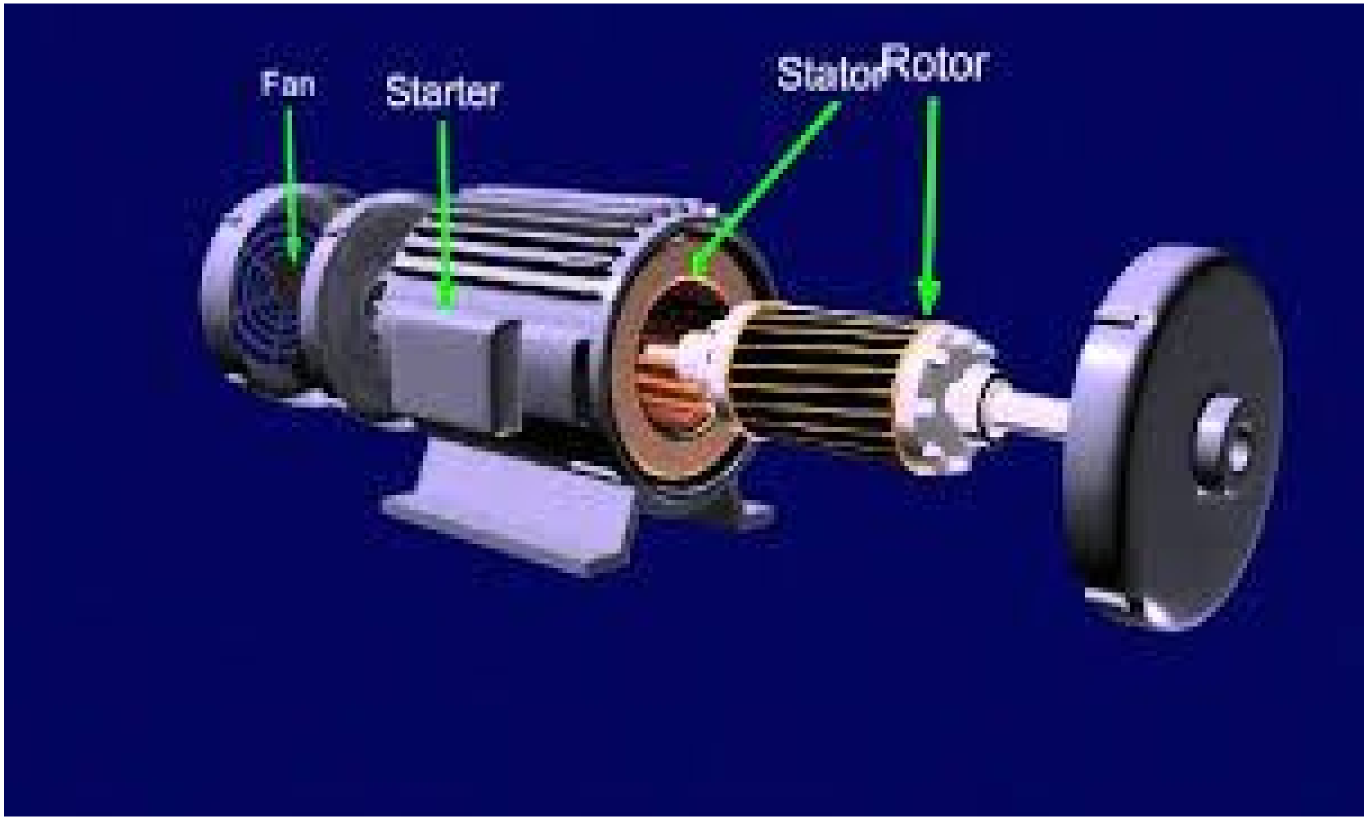
All day efficiency

$$\text{ordinary commercial efficiency} = \frac{\text{out put in watts}}{\text{input in watts}}$$

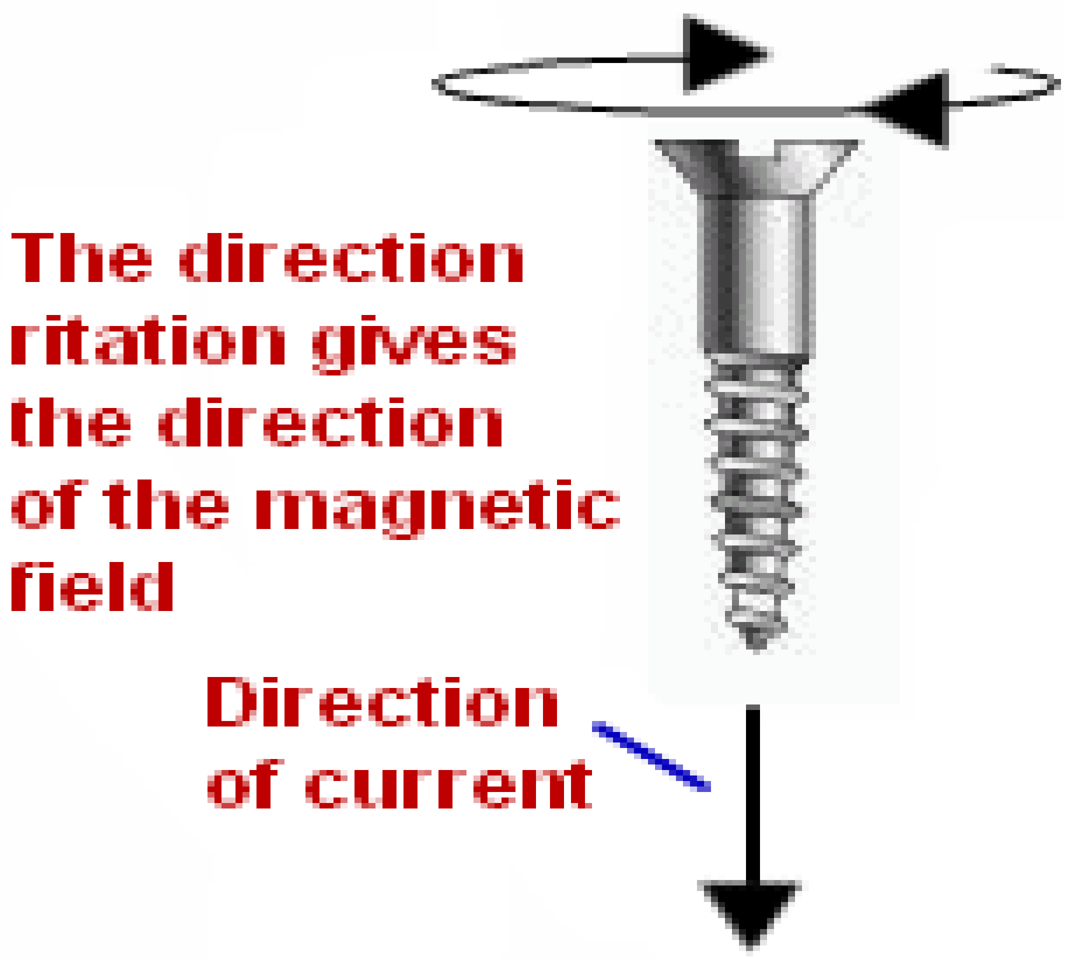
$$\eta_{all\ \text{day}} = \frac{\text{output in kWh}}{\text{Input in kWh}} (\mathit{for}\ 24\ \text{hours})$$

- All day efficiency is always less than the commercial efficiency

UNIT –II DC MACHINES



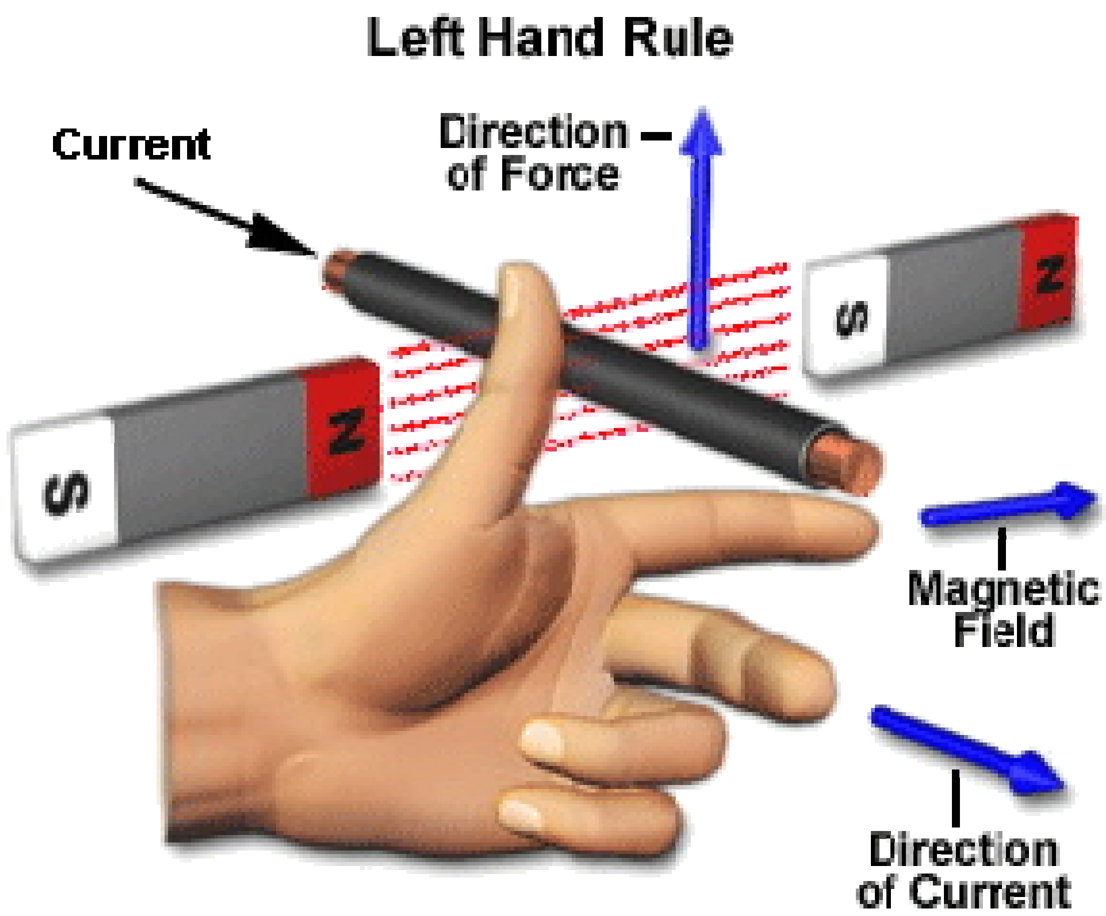
Maxwell's Cork screw Rule :



Maxwell's Cork screw Rule :

Hold the cork screw in yr right of current. Then the direction in which the hand rotates will be the direction of magnetic lines of forcehand and rotate it in clockwise in such a way that it advances in the direction

Fleming’s left hand rule



Fleming’s left hand rule

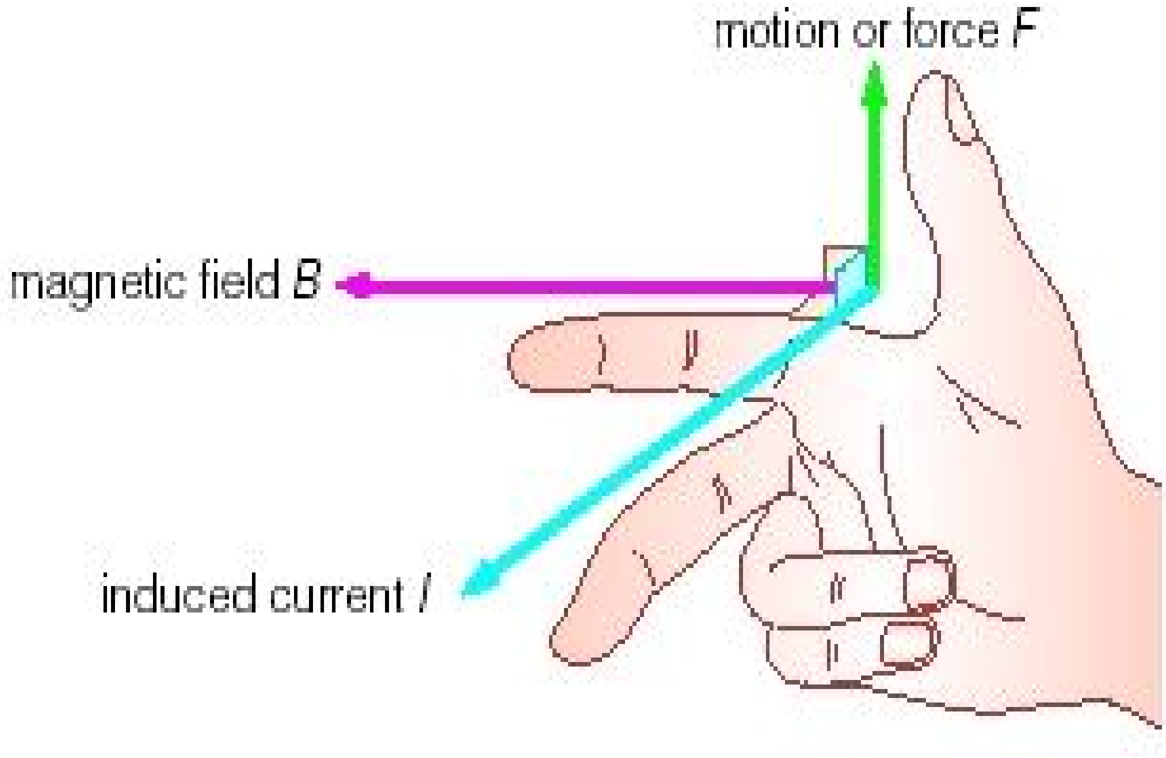
- Used to determine the <u>direction of force acting on a current carrying conductor</u> placed in a magnetic field

- The middle finger , the fore finger and thumb of the left hand are kept at right angles to one another .

  - The middle finger represent the direction of current
  - The fore finger represent the direction of magnetic field
  - The thumb will indicate the direction of force acting on the conductor .

This rule is used in motors.

Fleming's Right hand rule



Fleming’s Right hand rule

- Used to determine the direction of emf induced in a conductor
- The middle finger , the fore finger and thumb of the left hand are kept at right angles to one another.

- The fore finger represent the direction of magnetic field

- The thumb represent the direction of motion of the conductor

- The middle finger will indicate the direction of the inducted emf .

This rule is used in DC Generators

Len’s Law

The direction of induced emf is given by Lenz’s law .

According to this law, the induced emf will be acting in such a way so as to oppose the very cause of production of it .

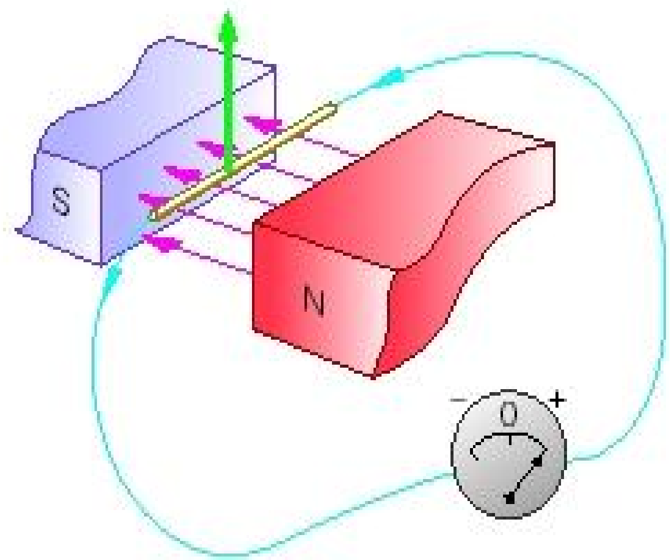
- $$e = -N\,(d\text{Ø}/dt)\ \text{volts}$$

DC Generator

Mechanical energy is converted to electrical energy

Three requirements are essential

1. Conductors
2. Magnetic field
3. Mechanical energy



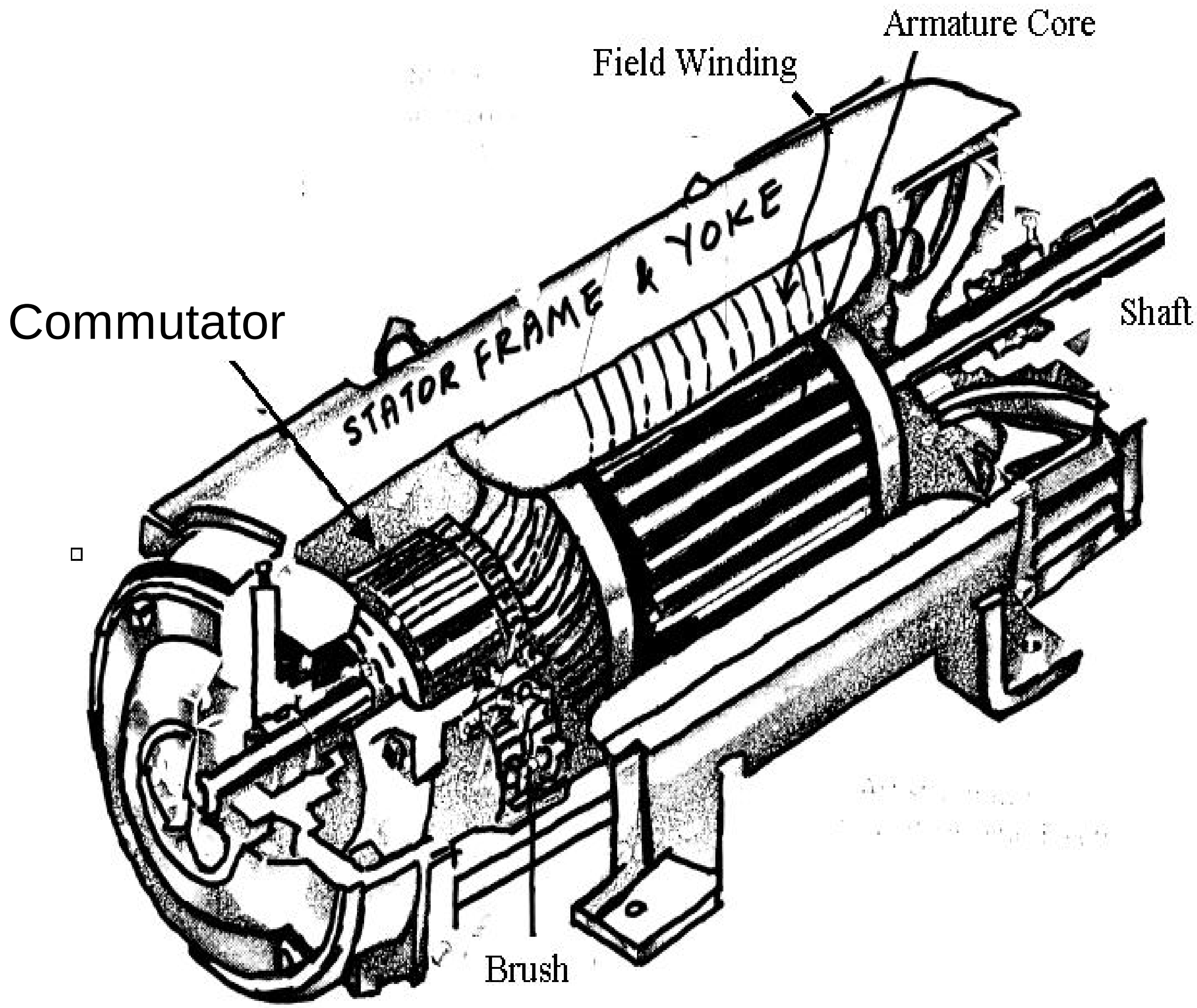
Working principle

- A generator works on the principles of Faraday’s law of electromagnetic induction

- Whenever a conductor is moved in the magnetic field , an emf is induced and the magnitude of the induced emf is directly proportional to the rate of change of flux linkage.

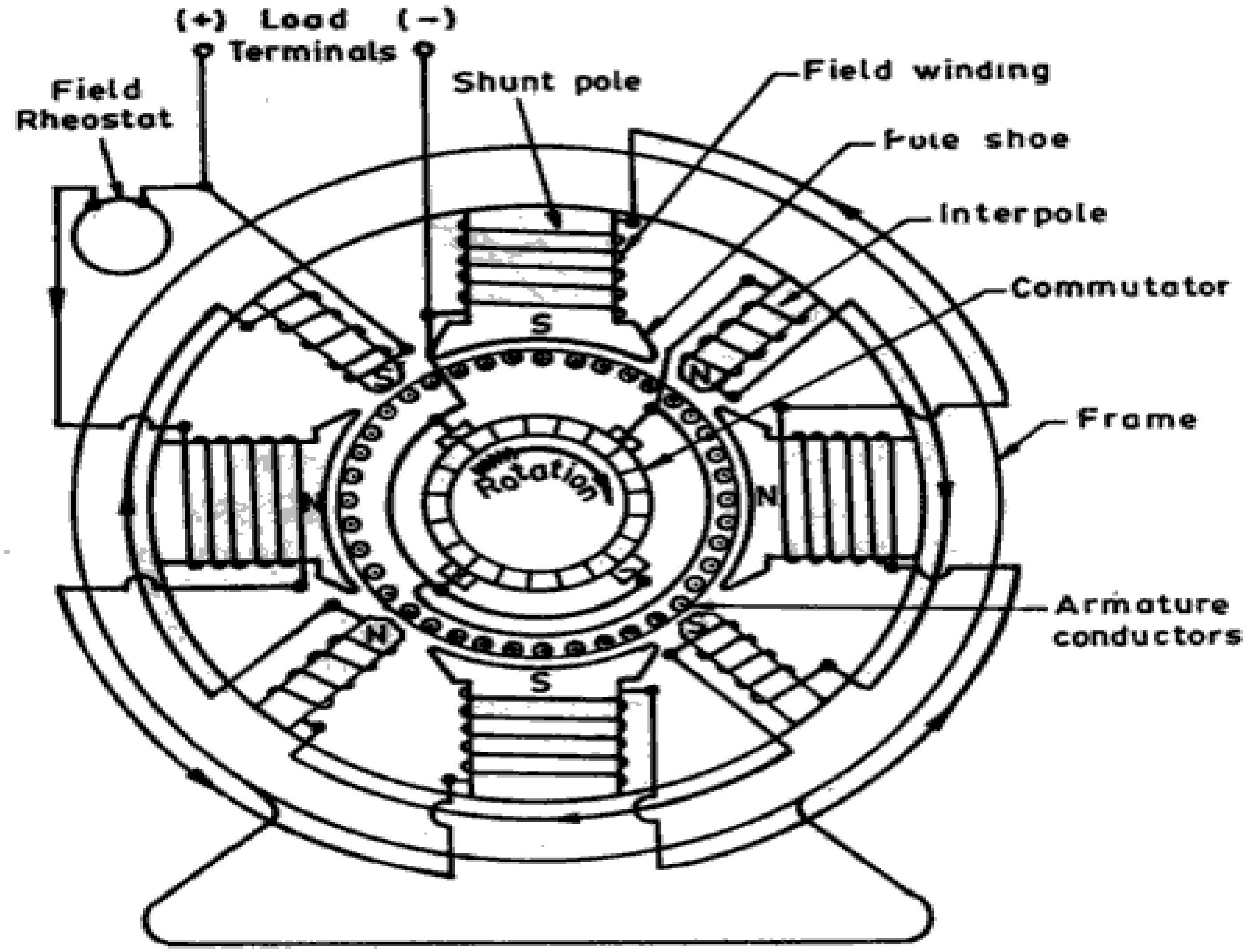
- This emf causes a current flow if the conductor circuit is closed .

DC Machine



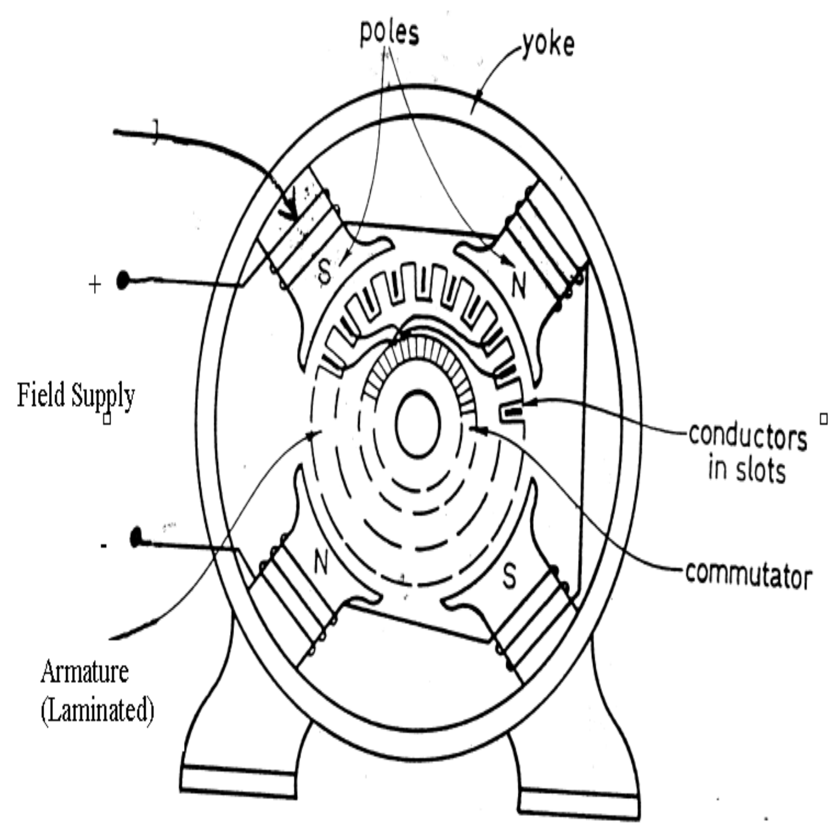
Armature Core
Field Winding
STATOR FRAME & YOKE
Commutator
Shaft
Brush

Sectional view of a DC machine



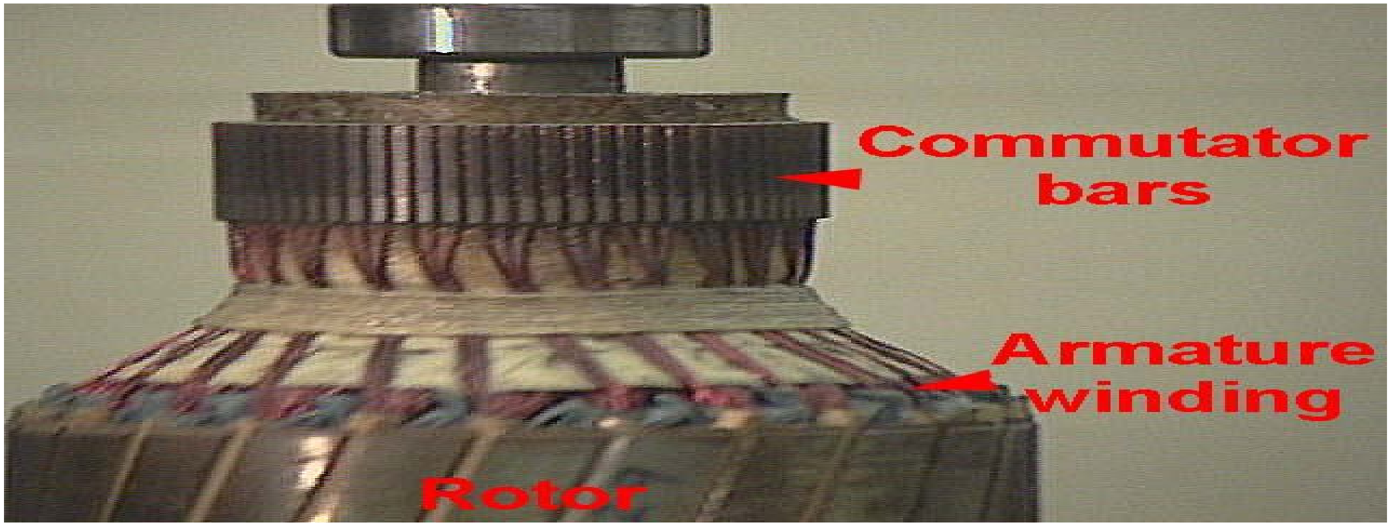
Construction of DC Generator

- Field system
- Armature core
- Armature winding
- Commutator
- Brushes

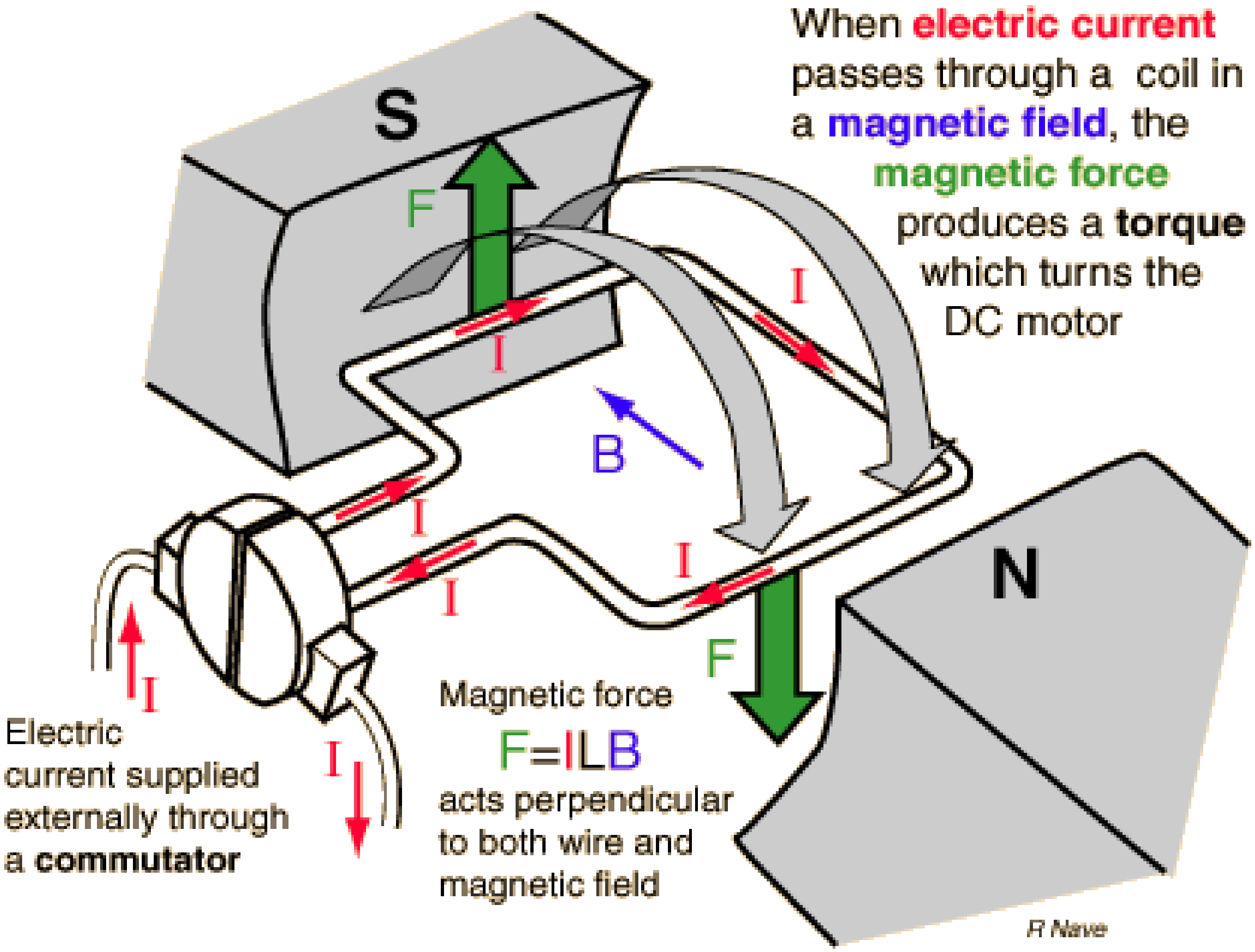


Field winding

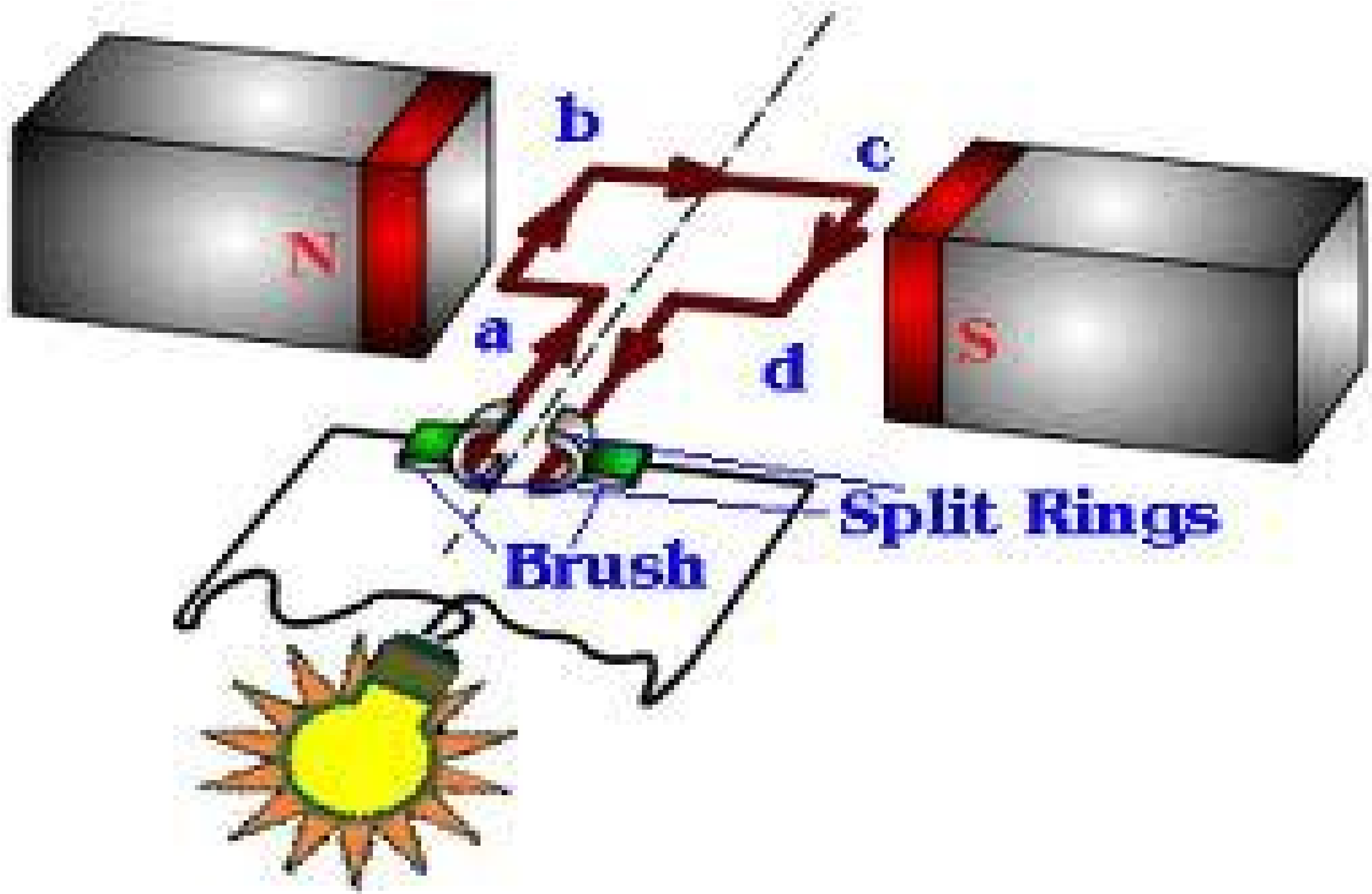
Rotor and rotor winding



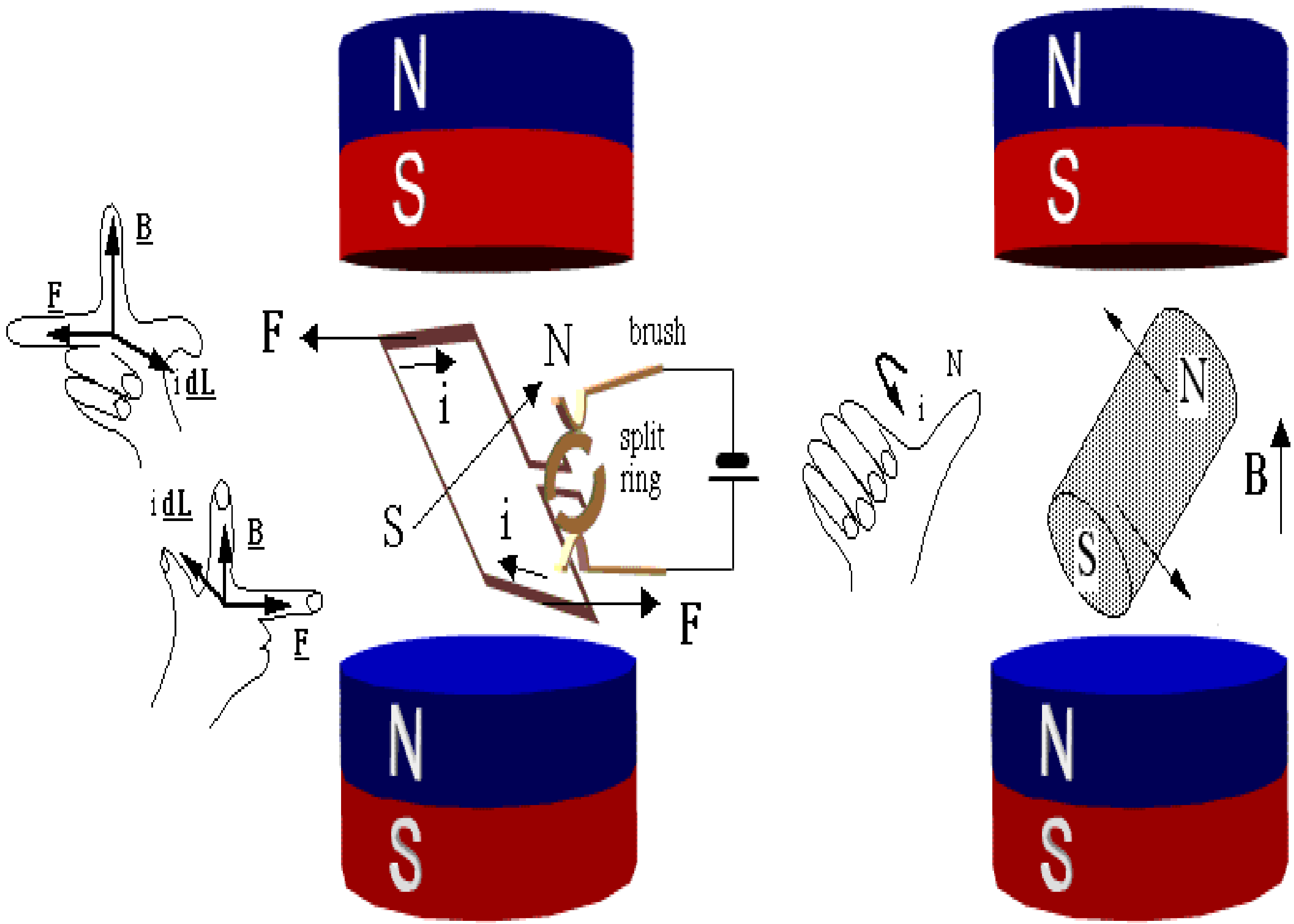
Working principle of DC motor



Working principle of DC motor



Force in DC motor



Armature winding

There are 2 types of winding

Lap and Wave winding

Lap winding

- **A = P**
- The armature windings are divided into no. of sections equal to the no of poles

Wave winding

- **A = 2**
- It is used in low current output and high voltage.
- 2 brushes

Field system

- It is for uniform magnetic field within which the armature rotates.
- Electromagnets are preferred in comparison with permanent magnets
- They are cheap , smaller in size , produce greater magnetic effect and
- Field strength can be varied

Field system consists of the following parts

- Yoke
- Pole cores
- Pole shoes
- Field coils

Armature core

- The armature core is cylindrical
- High permeability silicon steel stampings
- Impregnated
- Lamination is to reduce the eddy current loss

Commutator

- Connect with external circuit
- Converts ac into unidirectional current
- Cylindrical in shape
- Made of wedge shaped copper segments
- Segments are insulated from each other
- Each commutator segment is connected to armature conductors by means of a cu strip called riser.
- No of segments equal to no of coils

Carbon brush

- Carbon brushes are used in DC machines because they are soft materials
- It does not generate spikes when they contact commutator
- To deliver the current thro armature
- Carbon is used for brushes because it has negative temperature coefficient of resistance
- Self lubricating , takes its shape , improving area of contact

Brush rock and holder

Carbon brush

- Brush leads (pig tails)
- Brush rocker (brush gear)
- Front end cover
- Rear end cover
- Cooling fan
- Bearing
- Terminal box

EMF equation

Let,

- Ø= flux per pole in weber
- Z = Total number of conductor
- P = Number of poles
- A = Number of parallel paths
- N =armature speed in rpm
- Eg = emf generated in any on of the parallel path

EMF equation

Flux cut by 1 conductor
in 1 revolution $= P * \varphi$
Flux cut by 1 conductor in
60 sec $= P \varphi N / 60$
Avg emf generated in 1
conductor $= P\varphi N/60$
Number of conductors in
each parallel path $= Z / A$

Eg $= P\varphi NZ/60A$

Types of DC Generator

DC generators are generally classified according to their method of excitation .

- Separately excited DC generator
- Self excited D C generator

Further classification of DC Generator

- Series wound generator
- Shunt wound generator
- Compound wound generator
  - Short shunt & Long shunt
  - Cumulatively compound

    &

    Differentially compound

Characteristics

- No load saturation characteristic (Eo/If)
- Internal or Total characteristic (E/ Ia)
- External characteristic (V/I)

Critical field resistance

For appreciable generation of emf, the field resistance must be always less certain resistance, that resistance is called as the critical resistance of the machine .

General terms used in Armature reaction

Magnetic neutral axis :

It is perpendicular to the lines of force between the two opposite adjacent poles.

Leading pole Tip (LPT) :

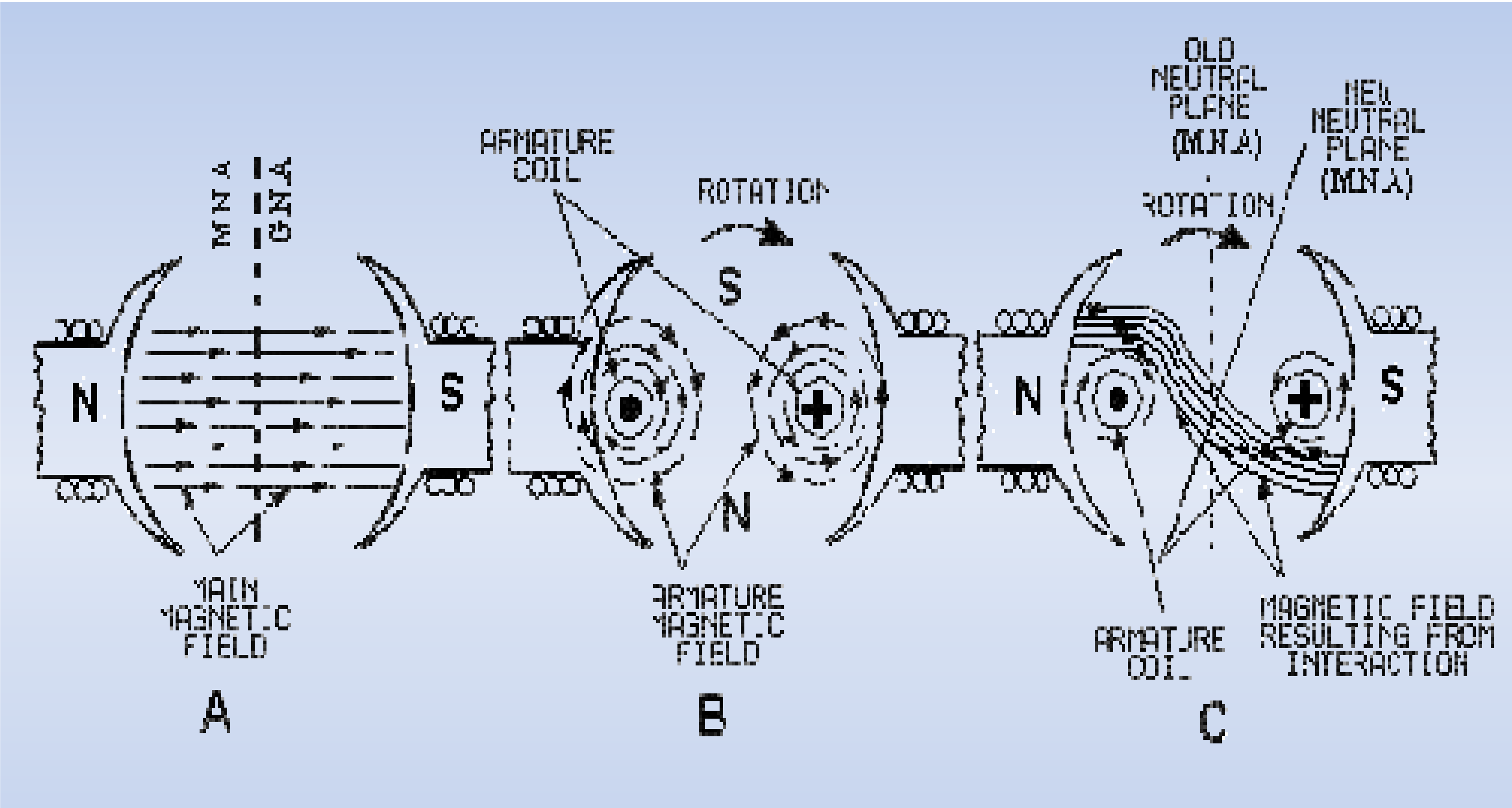
It is the end of the pole which first comes in contact with the armature.

Trailing pole tip :

It is the end of the pole which comes in contact later with the armature.

Armature Reaction

Interaction of Main field flux with Armature field flux



Effects of Armature Reaction

- It decreases the efficiency of the machine
- It produces sparking at the brushes
- It produces a demagnetising effect on the main poles
- It reduces the emf induced
- Self excited generators some times fail to build up emf

Armature reaction remedies

1. Brushes must be shifted to the new position of the MNA
2. Extra turns in the field winding
3. Slots are made on the tips to increase the reluctance
4. The laminated cores of the shoe are staggered
5. In big machines the compensating winding at pole shoes produces a flux which just opposes the armature mmf flux automatically

Commutation

- The change in direction of current takes place when the conductors are along the brush axis .
- During this reverse process brushes short circuit that coil and undergone commutation
- Due to this sparking is produced and the brushes will be damaged and also causes voltage dropping.

Losses in DC Generators

1. Copper losses or variable losses
2. Stray losses or constant losses

Stray losses : consist of (a) iron losses or core losses and (b) windage and friction losses .

Iron losses : occurs in the core of the machine due to change of magnetic flux in the core . Consist of hysteresis loss and eddy current loss.

Hysteresis loss depends upon the

Losses

Hysteresis loss depends upon the frequency , Flux density , volume and type of the core .

Eddy current losses : directly proportional to the flux density , frequency , thickness of the lamination .

Windage and friction losses are constant due to the opposition of wind and friction .

Applications

<u>Shunt Generators:</u>

a. in electro plating

b. for battery recharging

c. as exciters for AC generators.

<u>Series Generators :</u>

A. As boosters

B. As lighting arc lamps

DC Motors

<u>Converts Electrical energy into Mechanical energy</u>

Construction : Same for Generator and motor

Working principle : Whenever a current carrying conductor is placed in the magnetic field , a force is set up on the conductor.

Back emf

The induced emf in the rotating armature conductors always acts in the opposite direction of the supply voltage .

According to the Lenz's law, the direction of the induced emf is always so as to oppose the cause producing it .

In a DC motor , the supply voltage is the cause and hence this induced emf opposes the supply voltage.

Classification of DC motors

DC motors are mainly classified into three types as listed below:

- Shunt motor
- Series motor
- Compound motor
  - Differential compound
  - Cumulative compound

Torque

The turning or twisting force about an axis is called torque .

- $P = T * 2\pi N / 60$
- $E_b I_a = T_a * 2\pi N / 60$
- $T \propto \phi I_a$
- $T_a \propto I^2{}_a$

Characteristic of DC motors

- T/ Ia characteristic
- N/ I a characteristic
- N/T characteristic

Speed control of DC motors

According to the speed equation of a dc motor

$$N \propto Eb/\phi$$

$$\propto V- Ia\ Ra/\ \phi$$

Thus speed can be controlled by-

Flux control method: By Changing the flux by controlling the current through the field winding.

Armature control method: By Changing the armature resistance which in turn changes the voltage applied across the armature

Flux control

<u>Advantages of flux control:</u>

- It provides relatively smooth and easy control
- Speed control above rated speed is possible
- As the field winding resistance is high the field current is small. Power loss in the external resistance is small . Hence this method is economical

<u>Disadvantages:</u>

- Flux can be increased only upto its rated value
- High speed affects the commutation, motor operation becomes unstable

Armature voltage control method

- The speed is directly proportional to the voltage applied across the armature .
- Voltage across armature can be controlled by adding a variable resistance in series with the armature

Potential divider control :

If the speed control from zero to the rated speed is required , by rheostatic method then the voltage across the armature can be varied by connecting rheostat in a potential divider arrangement .

Starters for DC motors

Needed to limit the starting current .

1. Two point starter
2. Three point starter
3. Four point starter

Testing of DC machines

To determine the efficiency of as DC motor , the output and input should be known.

There are two methods.

- The load test or The direct method
- The indirect method

Direct method: In this method , the efficiency is determined by knowing the input and output power of the motor.

Indirect method: Swinburne's test is an indirect method of testing DC shunt machines to predetermine the effficency , as a motor and as a Generator. In this method, efficiency is calculated by determining the losses .

Applications:

<u>Shunt Motor</u>:

- Blowers and fans
- Centrifugal and reciprocating pumps
- Lathe machines
- Machine tools
- Milling machines
- Drilling machines

Applications:

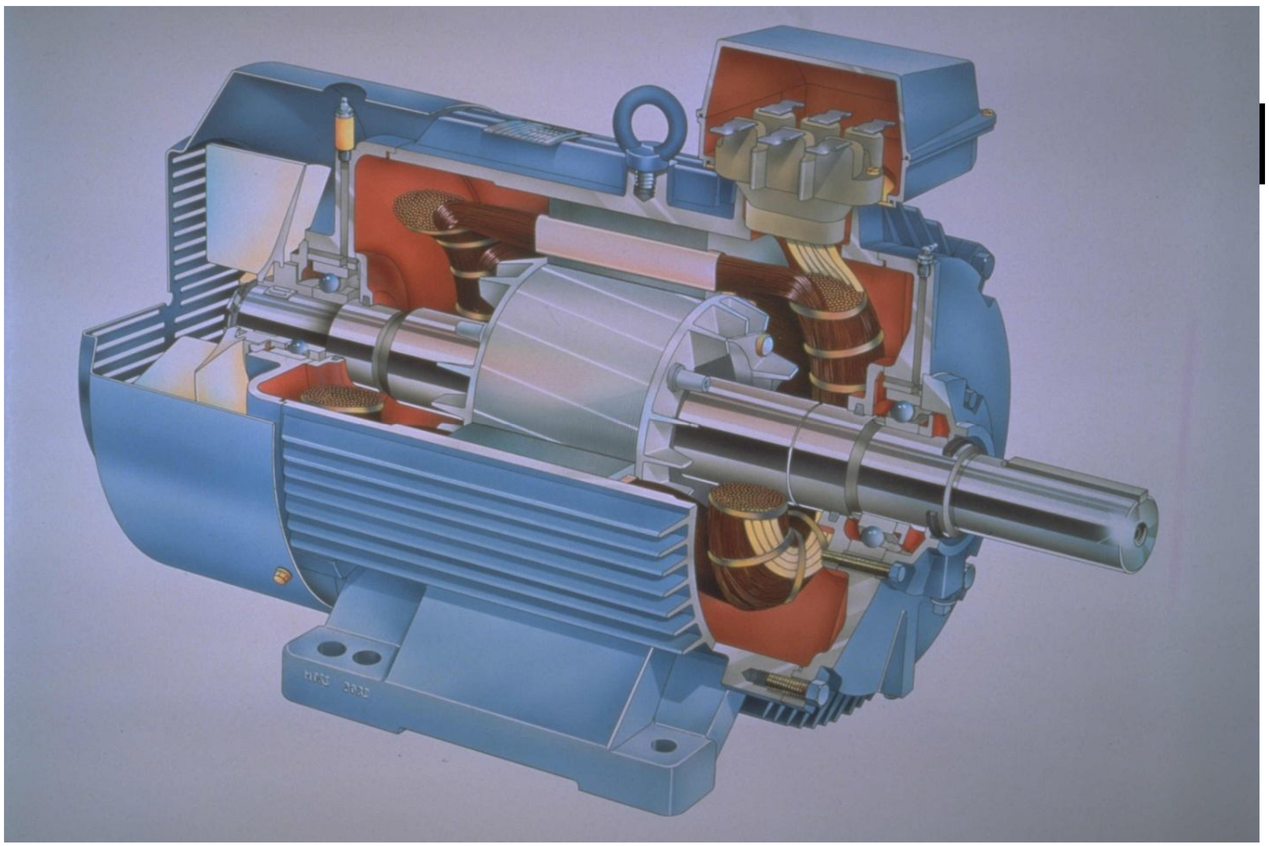
Series Motor:

- Cranes
- Hoists , Elevators
- Trolleys
- Conveyors
- Electric locomotives

Applications:

Cumulative compound Motor:

- Rolling mills
- Punches
- Shears
- Heavy planers
- Elevators

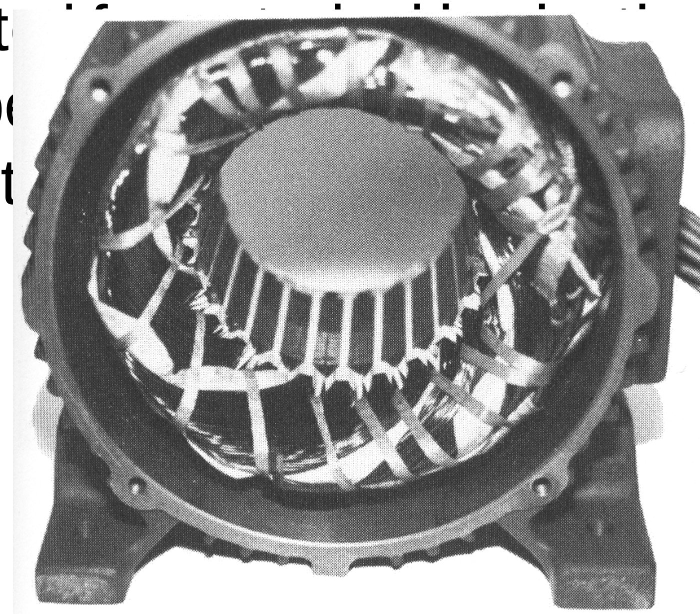


Introduction

- Three-phase induction motors are the most common and frequently encountered machines in industry
  - simple design, rugged, low-price, easy maintenance
  - wide range of power ratings: fractional horsepower to 10 MW
  - run essentially as constant speed from no-load to full load
  - Its speed depends on the frequency of the power source

Construction

- An induction motor has two main parts
  - a stationary stator
    - consisting of a steel frame that supports a hollow, cylindrical core
    - core, construct … s (why?), having a numb … roviding the space for the st …



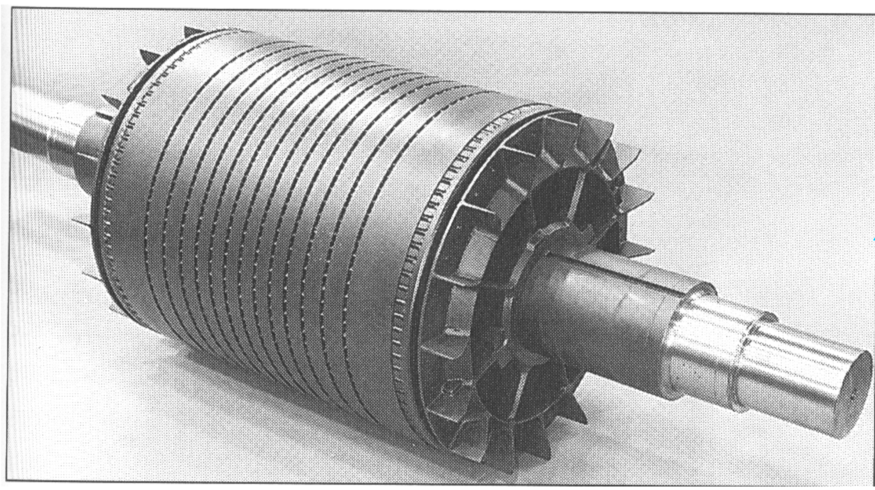
Stator of IM

Construction

- a revolving rotor
  - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
  - one of two types of rotor windings
  - conventional 3-phase windings made of insulated wire (wound-rotor) » similar to the winding on the stator
  - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)

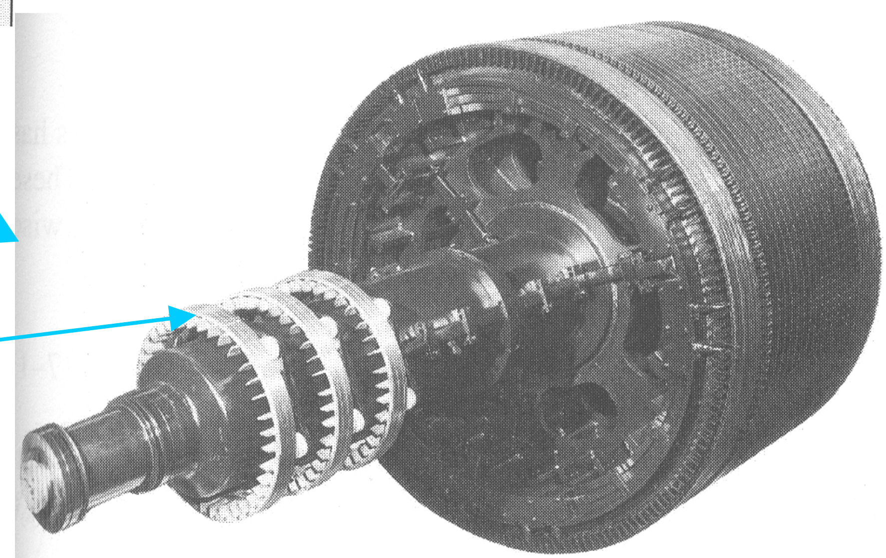
- Two basic design types depending on the rotor design
  - squirrel-cage: conducting bars laid into slots and shorted at both ends by shorting rings.
  - wound-rotor: complete set of three-phase windings exactly as the stator. Usually Y-connected, the ends of the three rotor wires are connected to 3 slip rings on the rotor shaft. In this way, the rotor circuit is accessible.

Construction



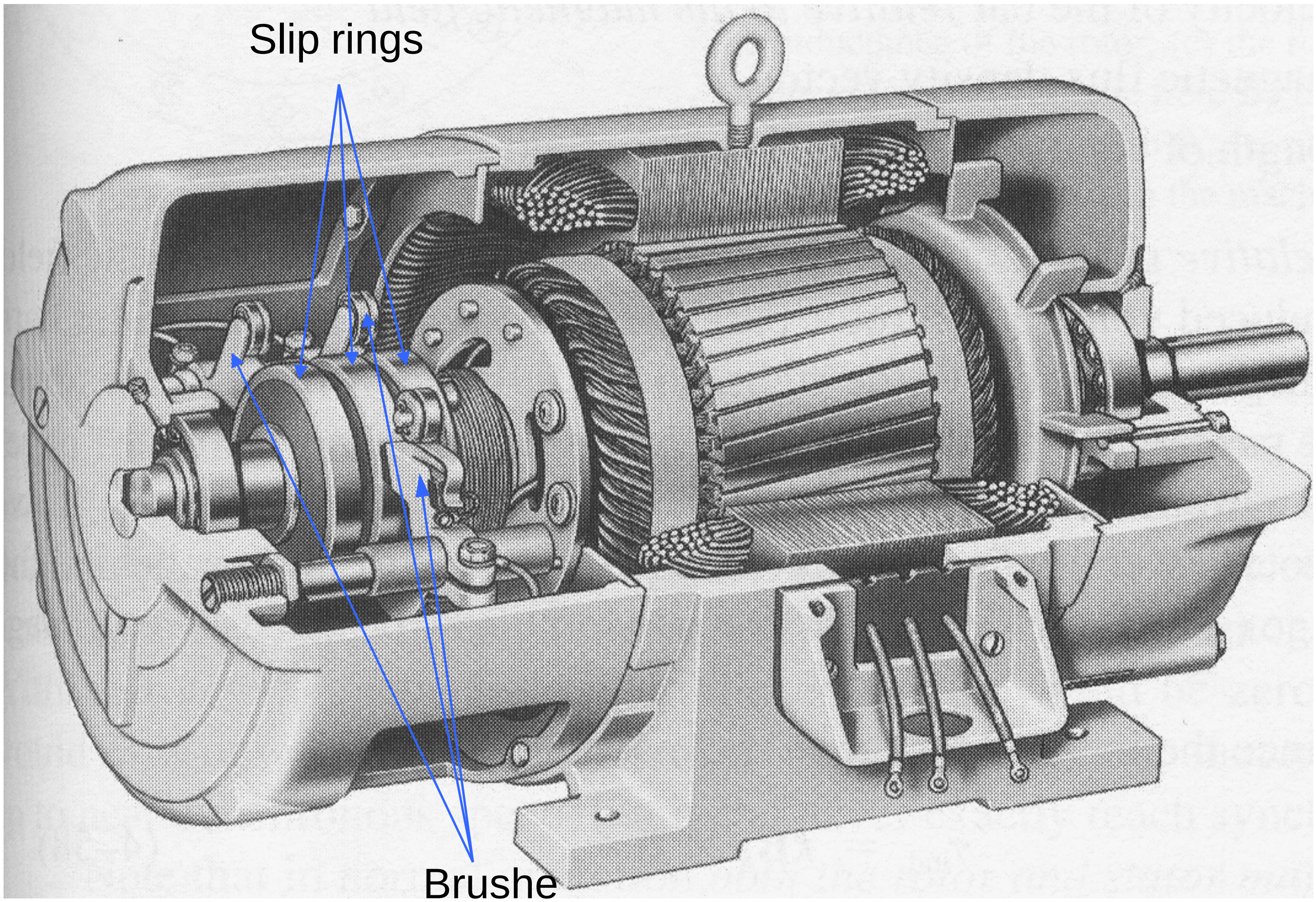
Squirrel cage rotor

Wound rotor



Notice the
slip rings

Construction



Cutaway in a typical wound-rotor IM. Notice the brushes and the slip rings

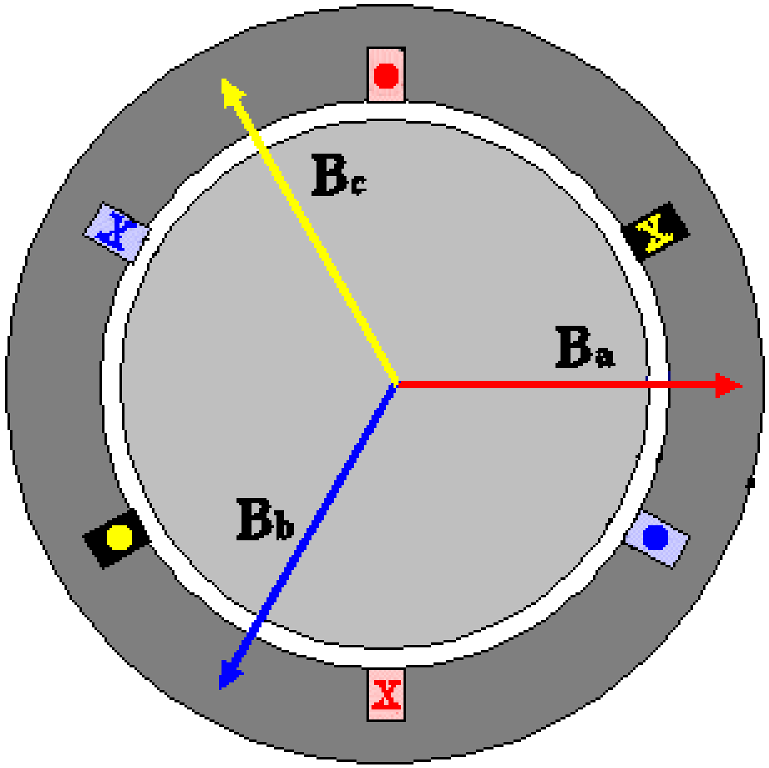
Rotating Magnetic Field

- Balanced three phase windings, i.e. mechanically displaced 120 degrees form each other, fed by balanced three phase source
- A rotating magnetic field with constant magnitude is produced, rotating with a speed

$$\boxed{n_{sync} = \frac{120\, f_e}{P} \quad rpm}$$

Where f_e is the supply frequency and

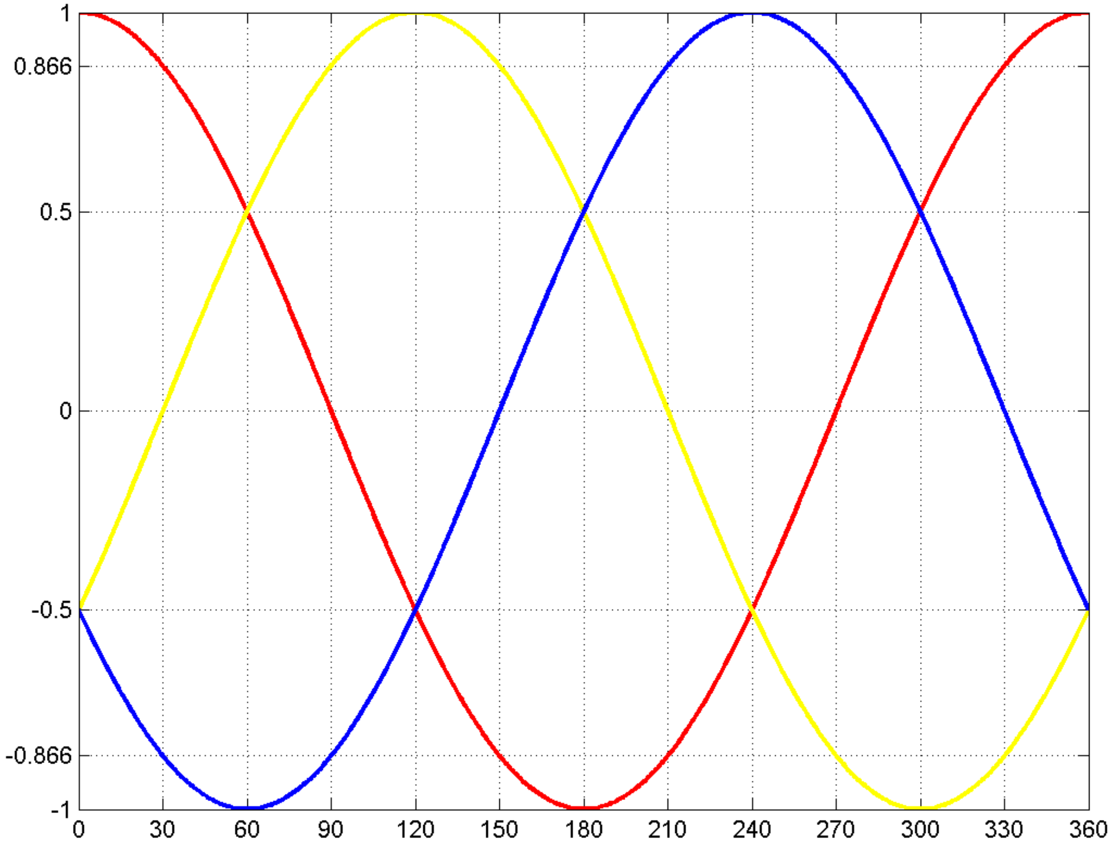
P is the no. of poles and n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)

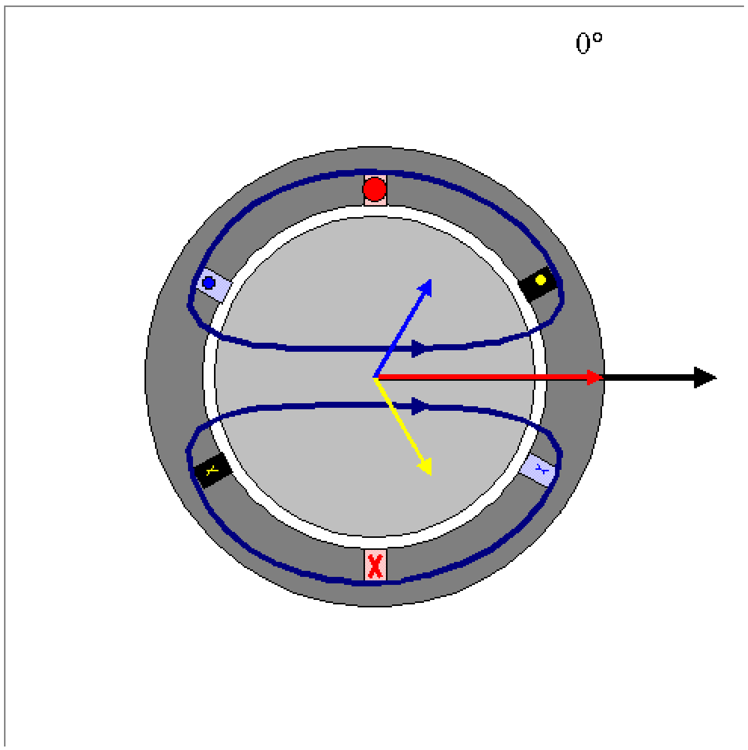


Synchronous speed

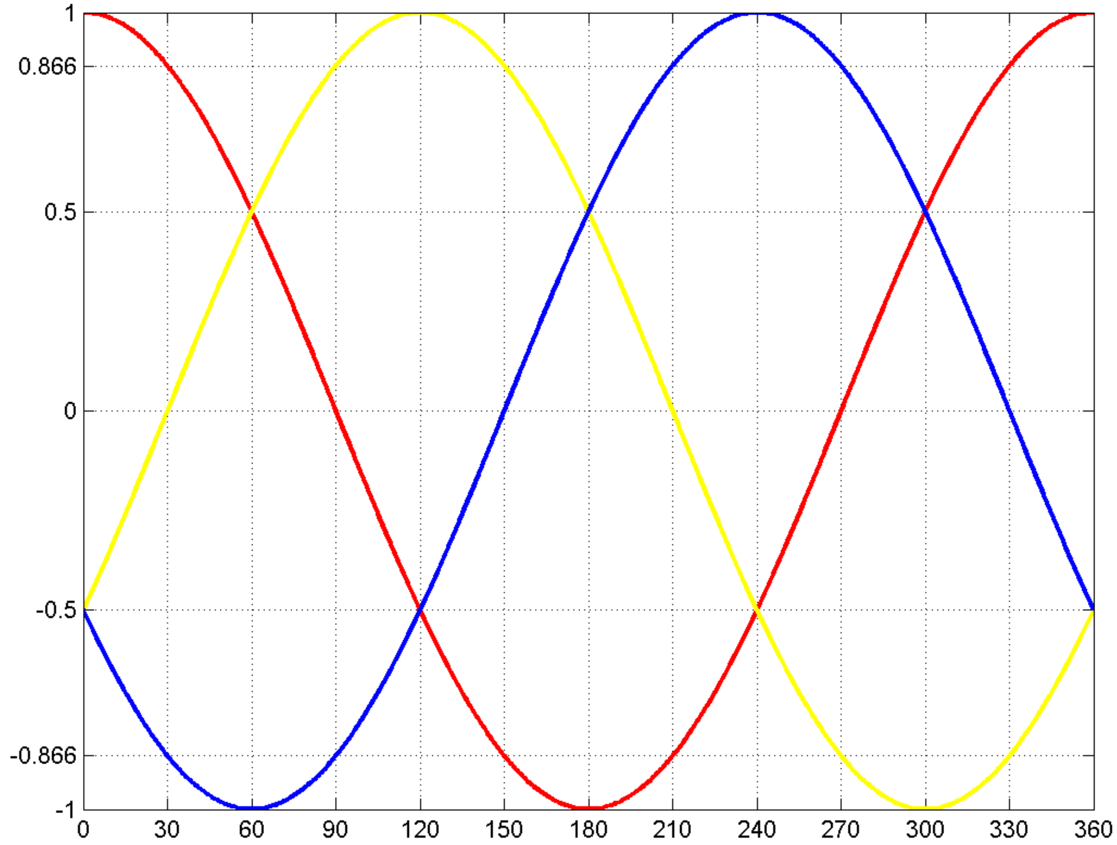
P	50 Hz	60 Hz
2	3000	3600
4	1500	1800
6	1000	1200
8	750	900
10	600	720
12	500	600

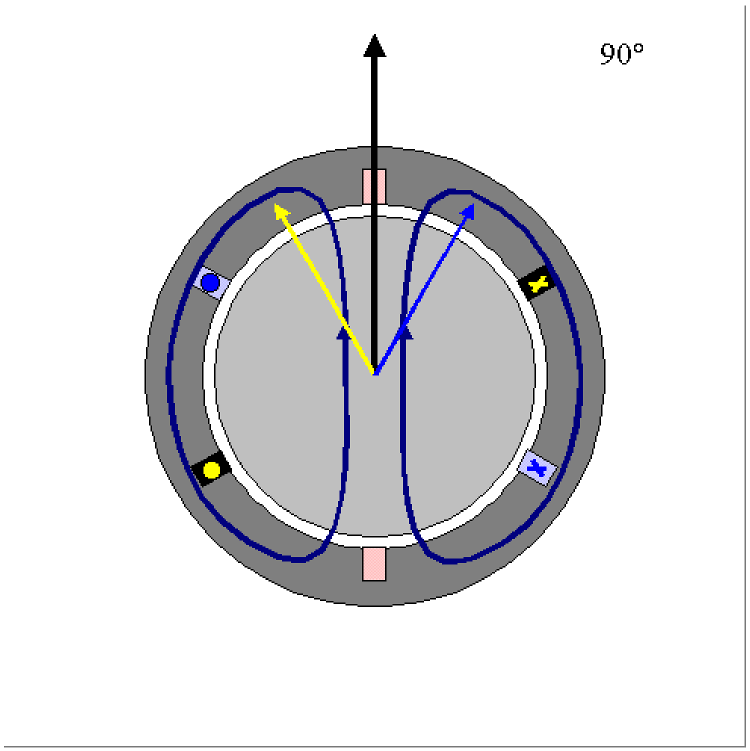
Rotating Magnetic Field





Rotating Magnetic Field





Rotating Magnetic Field

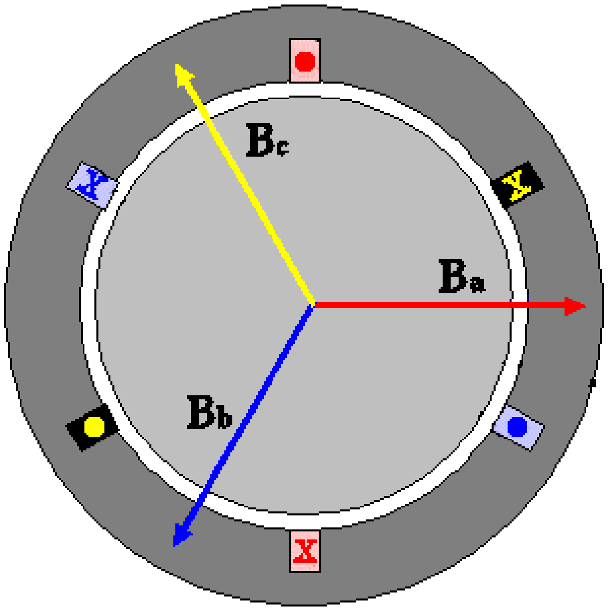
$$B_{net}(t) = B_a(t) + B_b(t) + B_c(t)$$

$$= B_M \sin(\omega t)\angle 0^\circ + B_M \sin(\omega t - 120^\circ)\angle 120^\circ + B_M \sin(\omega t - 240)\angle 240^\circ$$

$$= B_M \sin(\omega t)\hat{\mathbf{x}}$$

$$-[0.5 B_M \sin(\omega t - 120^\circ)]\hat{\mathbf{x}} - [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120^\circ)]\hat{\mathbf{y}}$$

$$-[0.5 B_M \sin(\omega t - 240^\circ)]\hat{\mathbf{x}} + [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240^\circ)]\hat{\mathbf{y}}$$



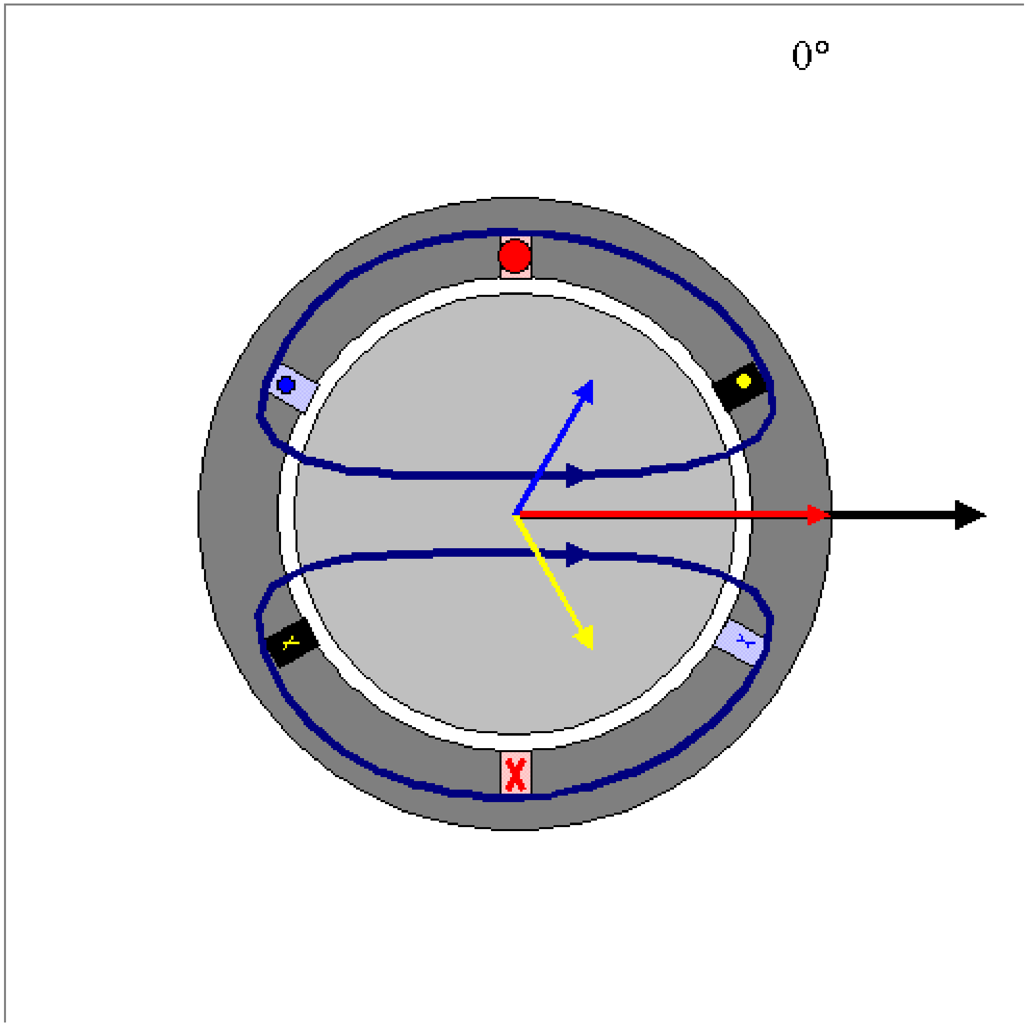
Rotating Magnetic Field

$$B_{net}(t) = [B_M \sin(\omega t) + \frac{1}{4} B_M \sin(\omega t) + \frac{\sqrt{3}}{4} B_M \cos(\omega t) + \frac{1}{4} B_M \sin(\omega t) - \frac{\sqrt{3}}{4} B_M \cos(\omega t)]\hat{\mathbf{x}}$$

$$+[-\frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t) + \frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t)]\hat{\mathbf{y}}$$

$$= [1.5 B_M \sin(\omega t)]\hat{\mathbf{x}} - [1.5 B_M \cos(\omega t)]\hat{\mathbf{y}}$$

Rotating Magnetic Field



Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

- At what speed will the IM run?
  - Can the IM run at the synchronous speed, why?
  - If rotor runs at the synchronous speed, which is the same speed of the rotating magnetic field, then the rotor will appear stationary to the rotating magnetic field and the rotating magnetic field will not cut the rotor. So, no induced current will flow in the rotor and no rotor magnetic flux will be produced so no torque is generated and the rotor speed will fall below the synchronous speed
  - When the speed falls, the rotating magnetic field

Induction motor speed

- So, the IM will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *Slip*

$$\boxed{n_{slip} = n_{sync} - n_m}$$

Where n_{slip}= slip speed

n_{sync}= speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$s = \frac{n_{sync} - n_m}{n_{sync}}$$

Where *s* is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn't have units

Induction Motors and Transformers

- Both IM and transformer works on the principle of induced voltage
  - Transformer: voltage applied to the primary windings produce an induced voltage in the secondary windings
  - Induction motor: voltage applied to the stator windings produce an induced voltage in the rotor windings
  - The difference is that, in the case of the induction motor, the secondary windings can move
  - Due to the rotation of the rotor (the secondary

Frequency

- The frequency of the voltage induced in the rotor is given by

$$f_r = \frac{P \times n}{120}$$

Where f_r = the rotor frequency (Hz)

P = number of stator poles

n = slip speed (rpm)

$$f_r = \frac{P \times (n_s - n_m)}{120}$$

$$= \frac{P \times sn_s}{120} = sf_e$$

Frequency

- What would be the frequency of the rotor's induced voltage at any speed n_m?

$$\boxed{f_r = s\ f_e}$$

- When the rotor is blocked (s=1) , the frequency of the induced voltage is equal to the supply frequency
- On the other hand, if the rotor runs at synchronous speed (s = 0), the frequency will be zero

Torque

- While the input to the induction motor is electrical power, its output is mechanical power and for that we should know some terms and quantities related to mechanical power
- Any mechanical load applied to the motor shaft will introduce a Torque on the motor shaft. This torque is related to the motor output power and the rotor speed

$$\tau_{load} = \frac{P_{out}}{\omega_m} \quad N.m$$

and

$$\omega_m = \frac{2\pi n_m}{60} \quad rad/s$$

Horse power

- Another unit used to measure mechanical power is the horse power
- It is used to refer to the mechanical output power of the motor
- Since we, as an electrical engineers, deal with watts as a unit to measure electrical power, there is a relation between horse power and watts

$$\boxed{hp = 746\ watts}$$

Example

A 208-V, 10hp, four pole, 60 Hz, Y-connected induction motor has a full-load slip of 5 percent

1. What is the synchronous speed of this motor?
2. What is the rotor speed of this motor at rated load?
3. What is the rotor frequency of this motor at rated load?
4. What is the shaft torque of this motor at rated load?

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120(60)}{4} = 1800\ rpm$$

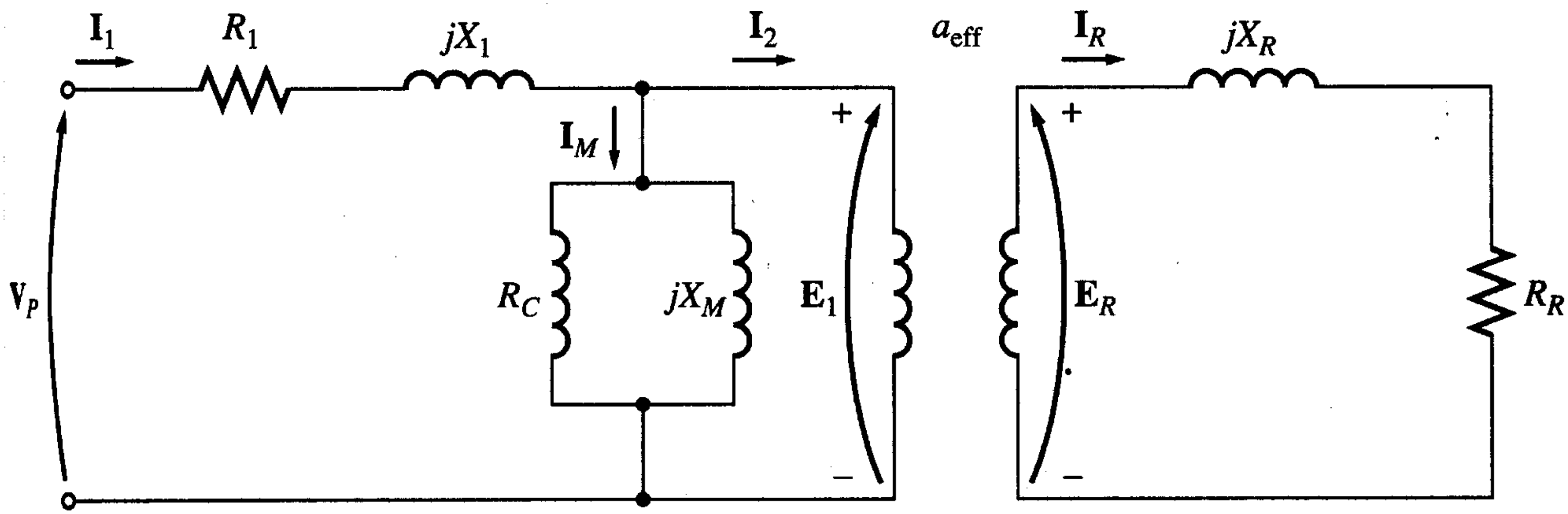
2. $$n_m = (1-s)n_s$$
$$= (1-0.05)\times 1800 = 1710\ rpm$$

3. $$f_r = sf_e = 0.05\times 60 = 3Hz$$

4. $$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{P_{out}}{2\pi \frac{n_m}{60}}$$
$$= \frac{10\,hp\times 746\,watt/hp}{1710\times 2\pi\times(1/60)} = 41.7\ N.m$$

Equivalent Circuit

- The induction motor is similar to the transformer with the exception that its secondary windings are free to rotate



As we noticed in the transformer, it is easier if we can combine these two circuits in one circuit but there

Equivalent Circuit

- When the rotor is locked (or blocked), i.e. s =1, the largest voltage and rotor frequency are induced in the rotor, Why?
- On the other side, if the rotor rotates at synchronous speed, i.e. s = 0, the induced voltage and frequency in the rotor will be equal to zero, Why?

$$\boxed{E_R = sE_{R0}}$$

Where E_{R0} is the largest value of the rotor's induced voltage obtained at s = 1(loacked rotor)

Equivalent Circuit

- The same is true for the frequency, i.e.

$$f_r = s\ f_e$$

- It is known that

$$X = \omega L = 2\pi f\ L$$

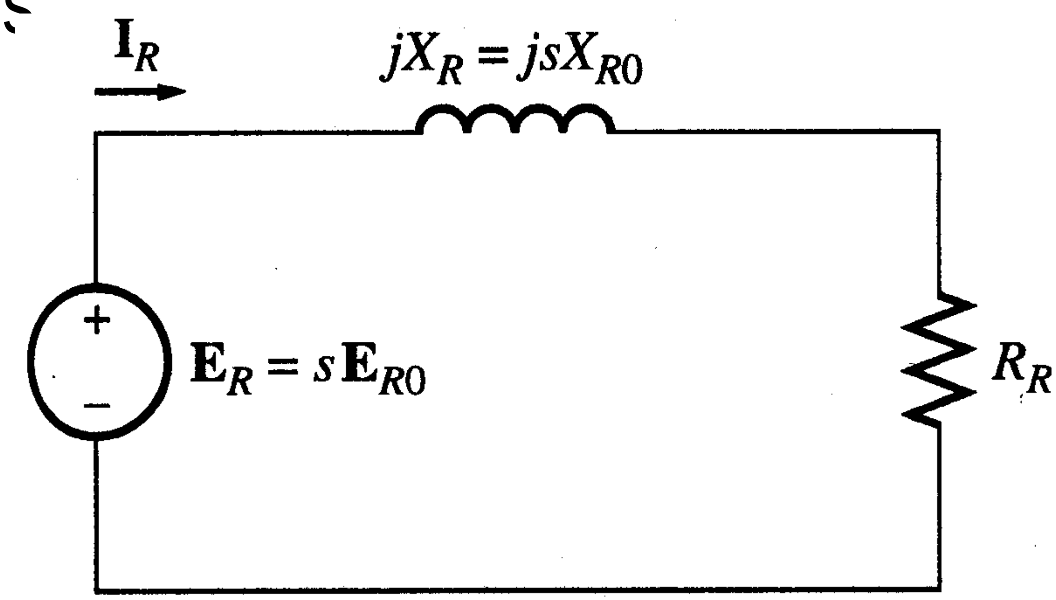
- So, as the frequency of the induced voltage in the rotor changes, the reactance of the rotor circuit also changes

$$\begin{aligned} X_r &= \omega_r L_r = 2\pi f_r L_r \\ &= 2\pi s f_e L_r \\ &= s X_{r0} \end{aligned}$$

Where X_{r0} is the rotor reactance
at the supply frequency
(at blocked rotor)

Equivalent Circuit

- Then, we can draw the rotor equivalent circuit as follows



Where E_R is the induced voltage in the rotor and R_R is the rotor resistance

Equivalent Circuit

- Now we can calculate the rotor current as

$$I_R = \frac{E_R}{(R_R + jX_R)}$$

$$= \frac{sE_{R0}}{(R_R + jsX_{R0})}$$

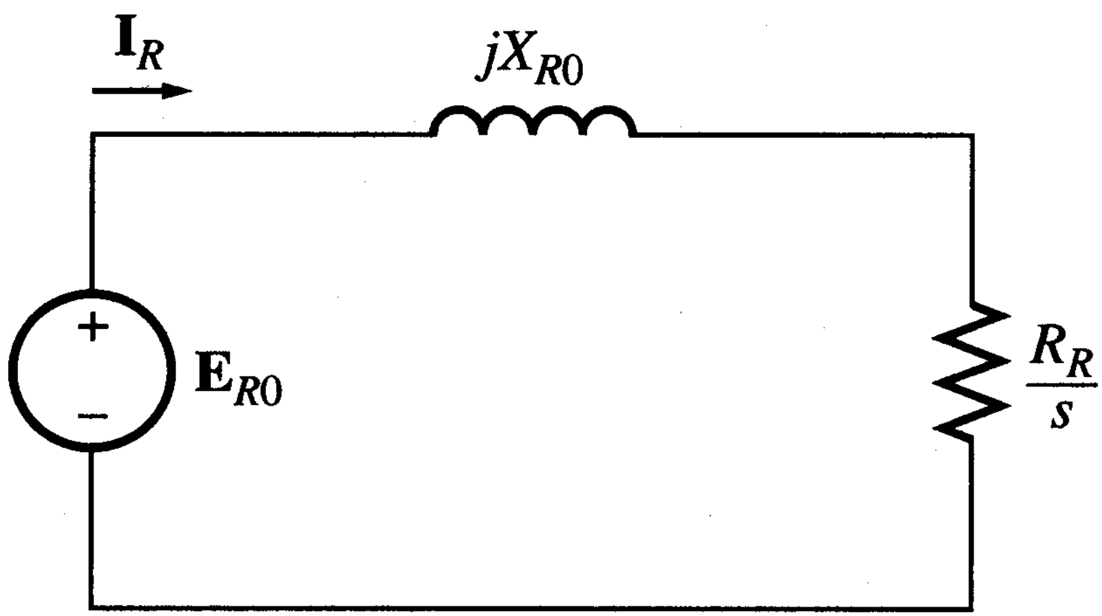
- Dividing both the numerator and denominator by *s* so nothing changes we get

$$I_R = \frac{E_{R0}}{(\frac{R_R}{s} + jX_{R0})}$$

Where E_{R0} is the induced voltage and X_{R0} is the rotor reactance at blocked rotor condition ($s = 1$)

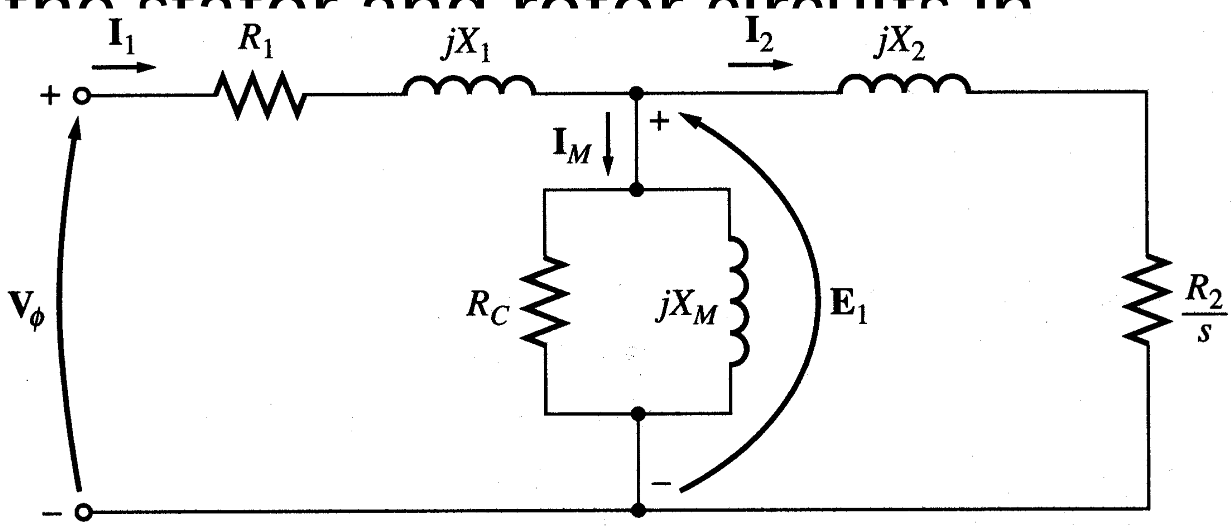
Equivalent Circuit

- Now we can have the rotor equivalent circuit



Equivalent Circuit

- Now as we managed to solve the induced voltage and different frequency problems, we can combine the stator and rotor circuits in one equivale



Where

$$X_2 = a_{eff}^2 X_{R0}$$

$$R_2 = a_{eff}^2 R_R$$

$$I_2 = \frac{I_R}{a_{eff}}$$

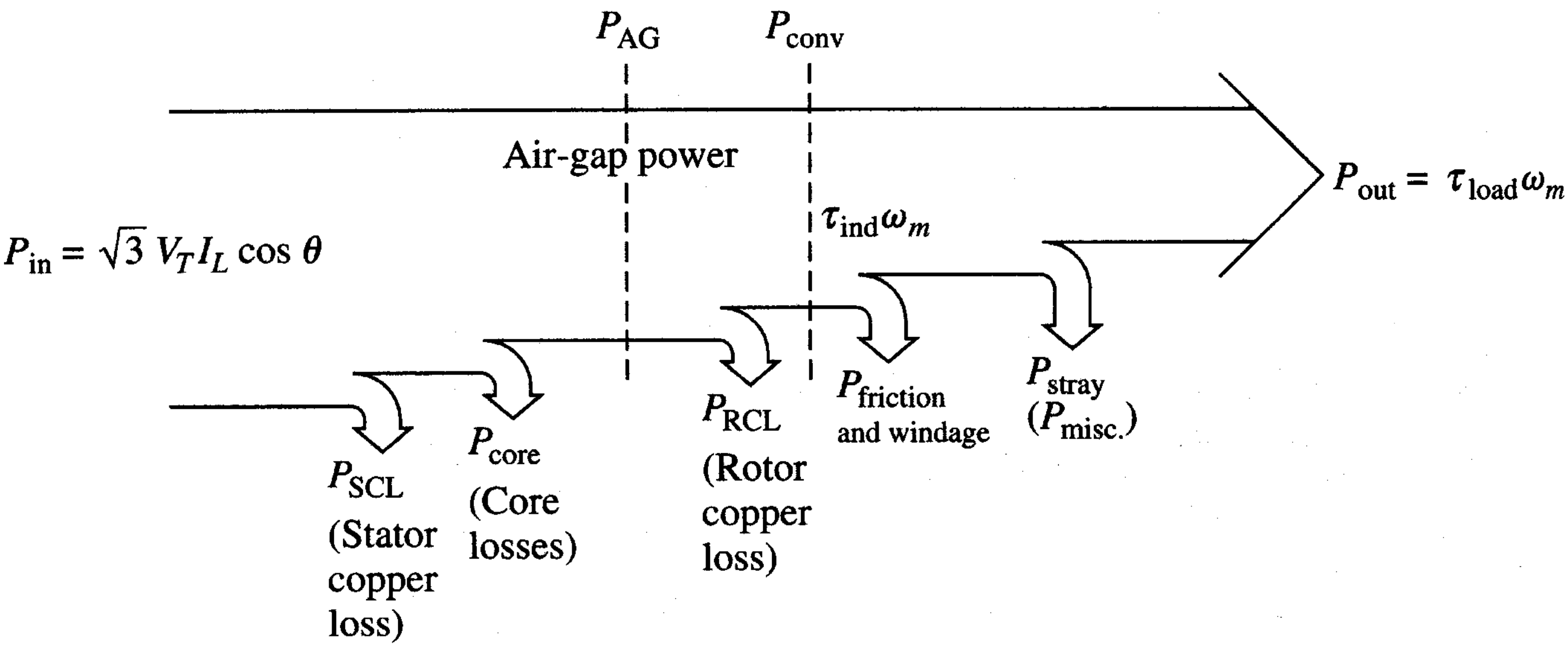
$$E_1 = a_{eff} E_{R0}$$

$$a_{eff} = \frac{N_S}{N_R}$$

Power losses in Induction machines

- Copper losses
  - Copper loss in the stator (P_{SCL}) = $I_1^2 R_1$
  - Copper loss in the rotor (P_{RCL}) = $I_2^2 R_2$
- Core loss (P_{core})
- Mechanical power loss due to friction and windage
- How this power flow in the motor?

Power flow in induction motor



Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

$$P_{RCL} = 3 I_2^2 R_2$$

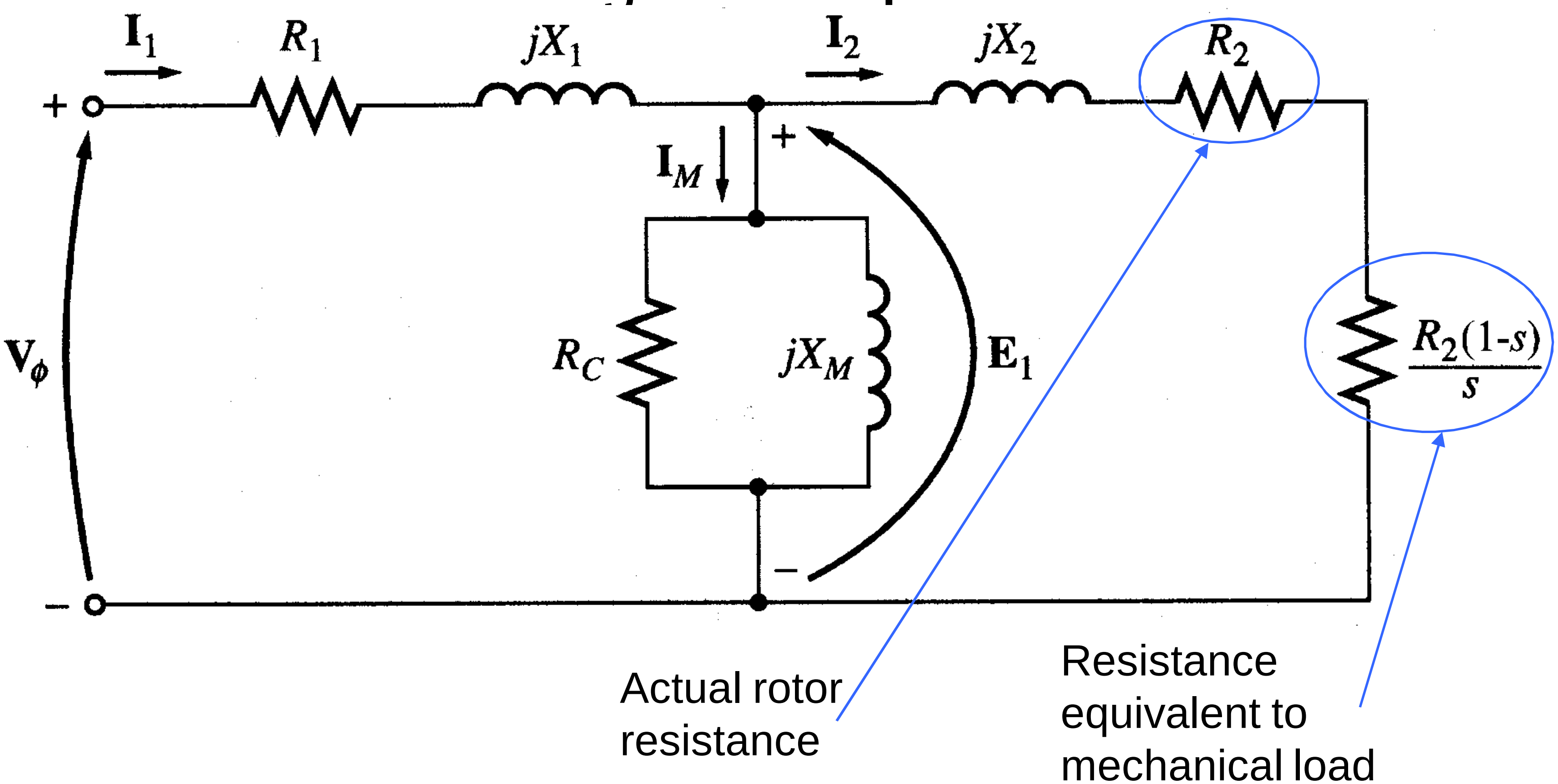
$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray})$$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

Equivalent Circuit

- We can rearrange the equivalent circuit as



Power relations

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core}) \quad = P_{conv} + P_{RCL} \quad = 3 I_2^2 \frac{R_2}{s} \quad = \frac{P_{RCL}}{s}$$

$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL} \quad = 3 I_2^2 \frac{R_2(1-s)}{s} \quad = \frac{P_{RCL}(1-s)}{s}$$

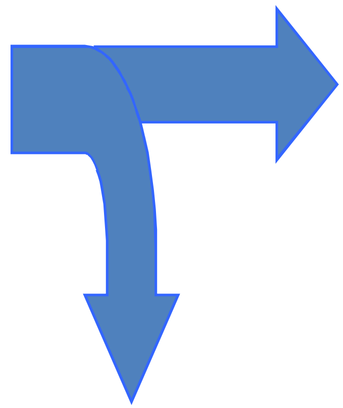
$$P_{conv} = (1-s) P_{AG}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray}) \qquad \tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_s}$$

Power relations

P_{AG}

1



P_{conv}

1-s

P_{RCL}

s

$$\boxed{\begin{array}{ccccc} P_{AG} & : & P_{RCL} & : & P_{conv} \\ 1 & : & s & : & 1-s \end{array}}$$

Example

A 480-V, 60 Hz, 50-hp, three phase induction motor is drawing 60A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

1. The air-gap power $P_{AG.}$
2. The power converted P_{conv}.
3. The output power P_{out}.
4. The efficiency of the motor

Solution

1.

$$P_{in} = \sqrt{3} V_L I_L \cos\theta$$

$$= \sqrt{3} \times 480 \times 60 \times 0.85 = 42.4 \text{ kW}$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$

$$= 42.4 - 2 - 1.8 = 38.6 \text{ kW}$$

2.

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= 38.6 - \frac{700}{1000} = 37.9 \text{ kW}$$

3.

$$P_{out} = P_{conv} - P_{F\&W}$$

$$= 37.9 - \frac{600}{1000} = 37.3 \text{ kW}$$

Solution

$$P_{out} = \frac{37.3}{0.746} = 50 \text{ hp}$$

4. $$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{37.3}{42.4} \times 100 = 88\%$$

Example

A 460-V, 25-hp, 60 Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$R_1 = 0.641\Omega$ $R_2 = 0.332\Omega$

$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$ $X_M = 26.3\ \Omega$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

1. Speed
2. Stator current
3. Power factor
4. P_{conv} and P_{out}
5. τ_{ind} and τ_{load}
6. Efficiency

Solution

1.
$$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 60}{4} = 1800 \text{ rpm}$$

$$n_m = (1-s)n_{sync} = (1-0.022) \times 1800 = 1760 \text{ rpm}$$

2.
$$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.332}{0.022} + j0.464$$

$$= 15.09 + j0.464 = 15.1\angle 1.76^\circ\ \Omega$$

$$Z_f = \frac{1}{1/jX_M + 1/Z_2} = \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$

$$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega$$

Solution

$$Z_{tot} = Z_{stat} + Z_f$$

$$= 0.641 + j1.106 + 12.94\angle 31.1°\ \Omega$$

$$= 11.72 + j7.79 = 14.07\angle 33.6°\ \Omega$$

$$I_1 = \frac{V_\phi}{Z_{tot}} = \frac{\frac{460\angle 0°}{\sqrt{3}}}{14.07\angle 33.6°} = 18.88\angle -33.6°\ \text{A}$$

$$PF = \cos 33.6° = 0.833 \quad \text{lagging}$$

3. $$P_{in} = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 460 \times 18.88 \times 0.833 = 12530\ \text{W}$$

4. $$P_{SCL} = 3 I_1^2 R_1 = 3(18.88)^2 \times 0.641 = 685\ \text{W}$$

$$P_{AG} = P_{in} - P_{SCL} = 12530 - 685 = 11845\ \text{W}$$

Solution

$$P_{conv} = (1-s)P_{AG} = (1-0.022)(11845) = 11585 \text{ W}$$

$$P_{out} = P_{conv} - P_{F\&W} = 11585 - 1100 = 10485 \text{ W}$$

$$= \frac{10485}{746} = 14.1 \text{ hp}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{11845}{2\pi \times 1800/60} = 62.8 \text{ N.m}$$

5.

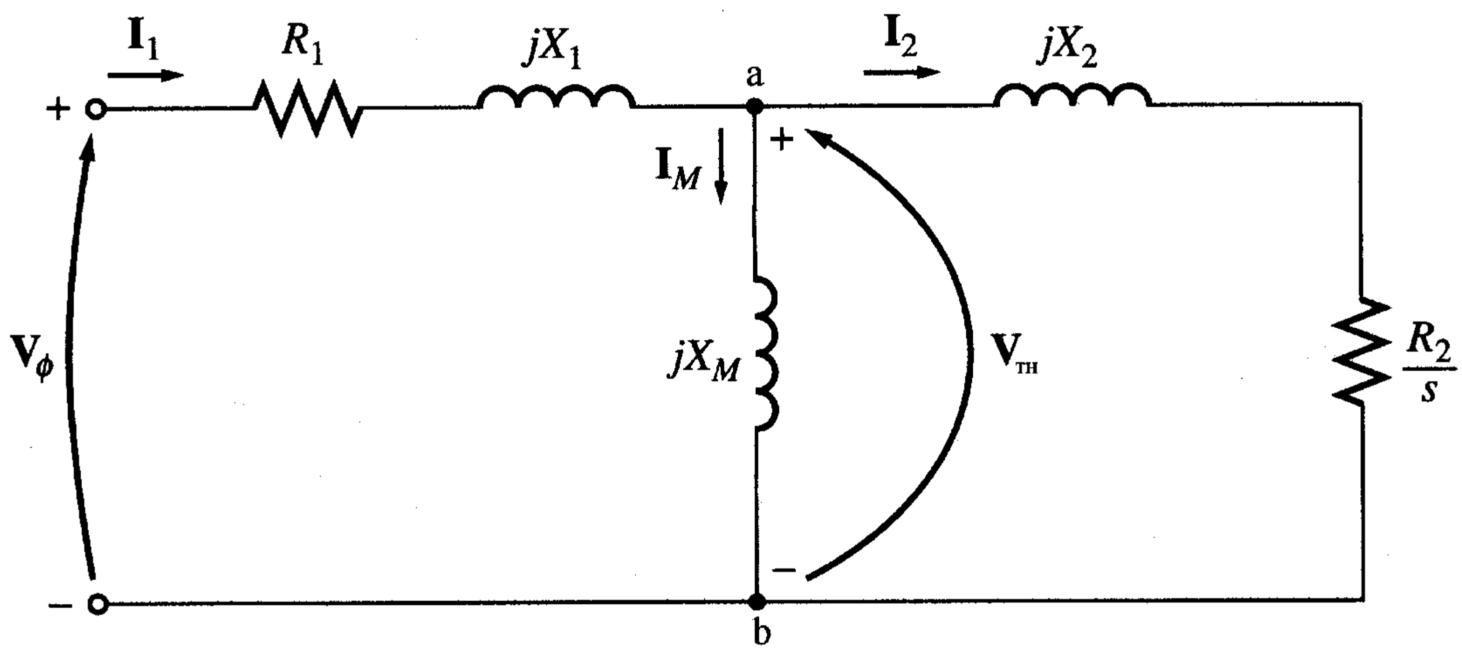
$$\tau_{load} = \frac{P_{out}}{\omega_m} = \frac{10485}{2\pi \times 1760/60} = 56.9 \text{ N.m}$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{10485}{12530} \times 100 = 83.7\%$$

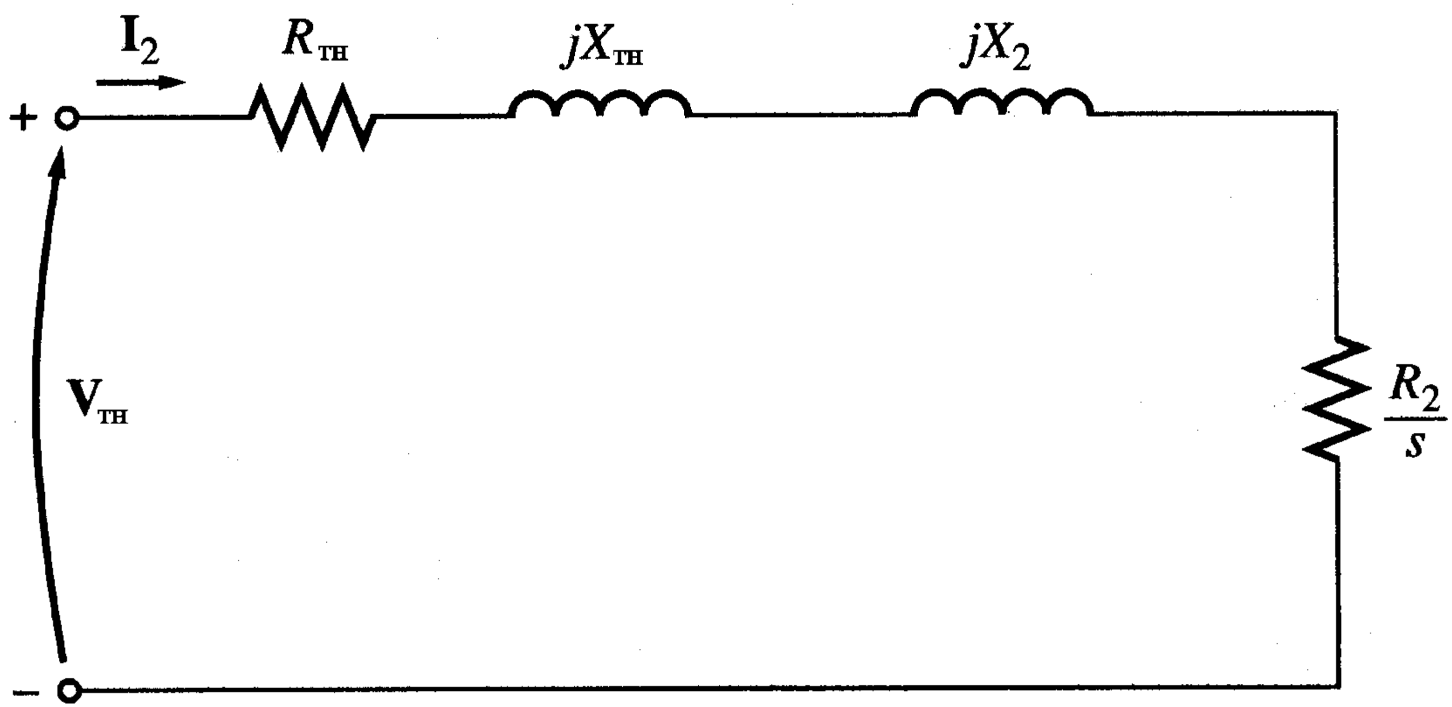
6.

Torque, power and Thevenin's Theorem

- Thevenin's theorem can be used to transform the network to the left of points 'a' and 'b' into an equivalent voltage source V_{TH} in series w



Torque, power and Thevenin's Theorem



$$V_{TH} = V_\phi \frac{jX_M}{R_1 + j(X_1 + X_M)} \qquad |V_{TH}| = |V_\phi| \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$R_{TH} + jX_{TH} = (R_1 + jX_1) // jX_M$$

Torque, power and Thevenin's Theorem

- Since $X_M >> X_1$ and $X_M >> R_1$

$$V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}$$

- Because $X_M >> X_1$ and $X_M + X_1 >> R_1$

$$R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2$$

$$X_{TH} \approx X_1$$

Torque, power and Thevenin's Theorem

$$I_2 = \frac{V_{TH}}{Z_T} = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R_2}{s}\right)^2 + (X_{TH} + X_2)^2}}$$

Then the power converted to mechanical (P_{conv})

$$P_{conv} = 3I_2^2 \frac{R_2(1-s)}{s}$$

And the internal mechanical torque (T_{conv})

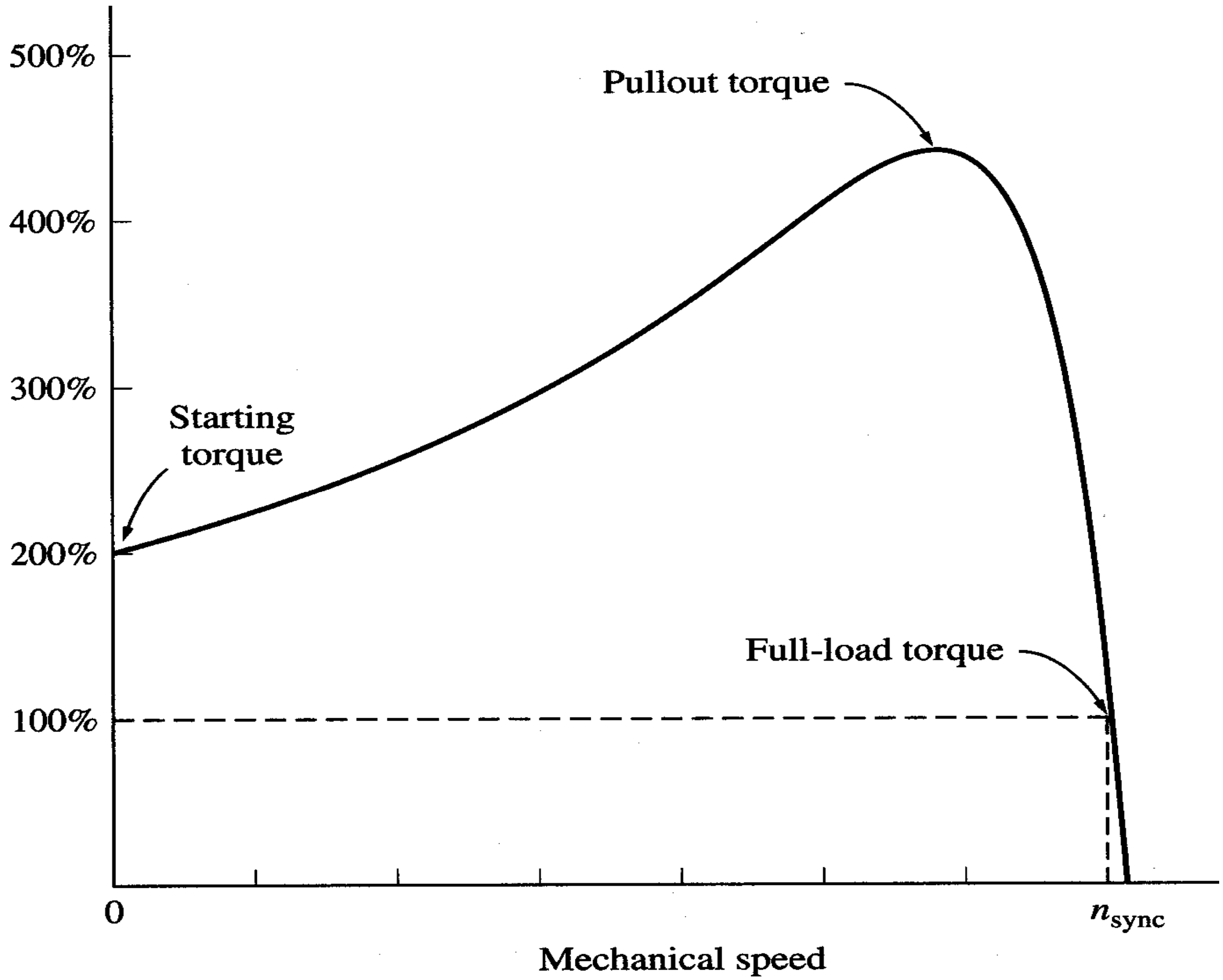
$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{conv}}{(1-s)\omega_s} = \frac{3I_2^2 \frac{R_2}{s}}{\omega_s} = \frac{P_{AG}}{\omega_s}$$

Torque, power and Thevenin's Theorem

$$\tau_{ind} = \frac{3}{\omega_s}\left(\frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R_2}{s}\right)^2 + (X_{TH} + X_2)^2}}\right)^2 \left(\frac{R_2}{s}\right)$$

$$\boxed{\tau_{ind} = \frac{1}{\omega_s}\frac{3V_{TH}^2\left(\frac{R_2}{s}\right)}{\left(R_{TH} + \frac{R_2}{s}\right)^2 + (X_{TH} + X_2)^2}}$$

Torque-speed characteristics



Typical torque-speed characteristics of induction motor

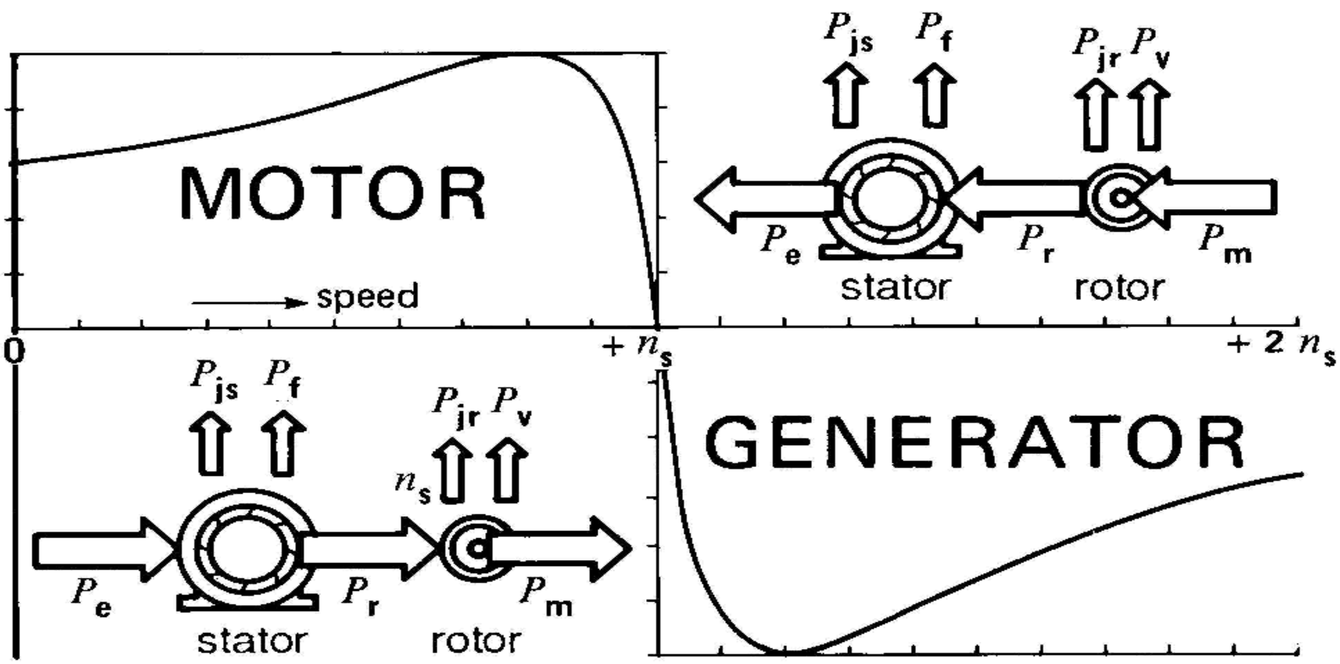
Comments

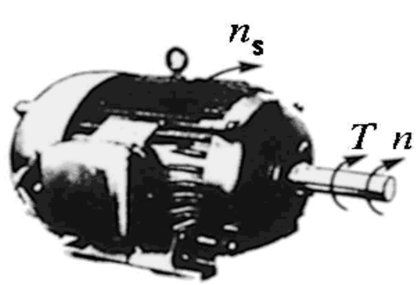
1. The induced torque is zero at synchronous speed. Discussed earlier.
2. The curve is nearly linear between no-load and full load. In this range, the rotor resistance is much greater than the reactance, so the rotor current, torque increase linearly with the slip.
3. There is a maximum possible torque that can't be exceeded. This torque is called *pullout torque* and is 2 to 3 times the rated

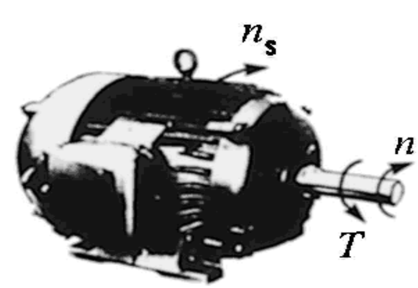
Comments

4. The starting torque of the motor is slightly higher than its full-load torque, so the motor will start carrying any load it can supply at full load.
5. The torque of the motor for a given slip varies as the square of the applied voltage.
6. If the rotor is driven faster than synchronous speed it will run as a generator, converting mechanical power to electric power.

Complete Speed-torque c/c







Maximum torque

- Maximum torque occurs when the power transferred to R_2/s is maximum.
- This condition occurs when R_2/s equals the magnitude of the impedance $R_{TH} + j\,(X_{TH} + X_2)$

$$\frac{R_2}{s_{T_{\max}}} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}$$

$$\boxed{s_{T_{\max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}}$$

Maximum torque

- The corresponding maximum torque of an induction motor equals

$$\boxed{\tau_{\max} = \frac{1}{2\omega_s}\left(\frac{3V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}\right)}$$

The slip at maximum torque is directly proportional to the rotor resistance R_2

The maximum torque is independent of R_2

Maximum torque

- Rotor resistance can be increased by inserting external resistance in the rotor of a wound-rotor induction motor.

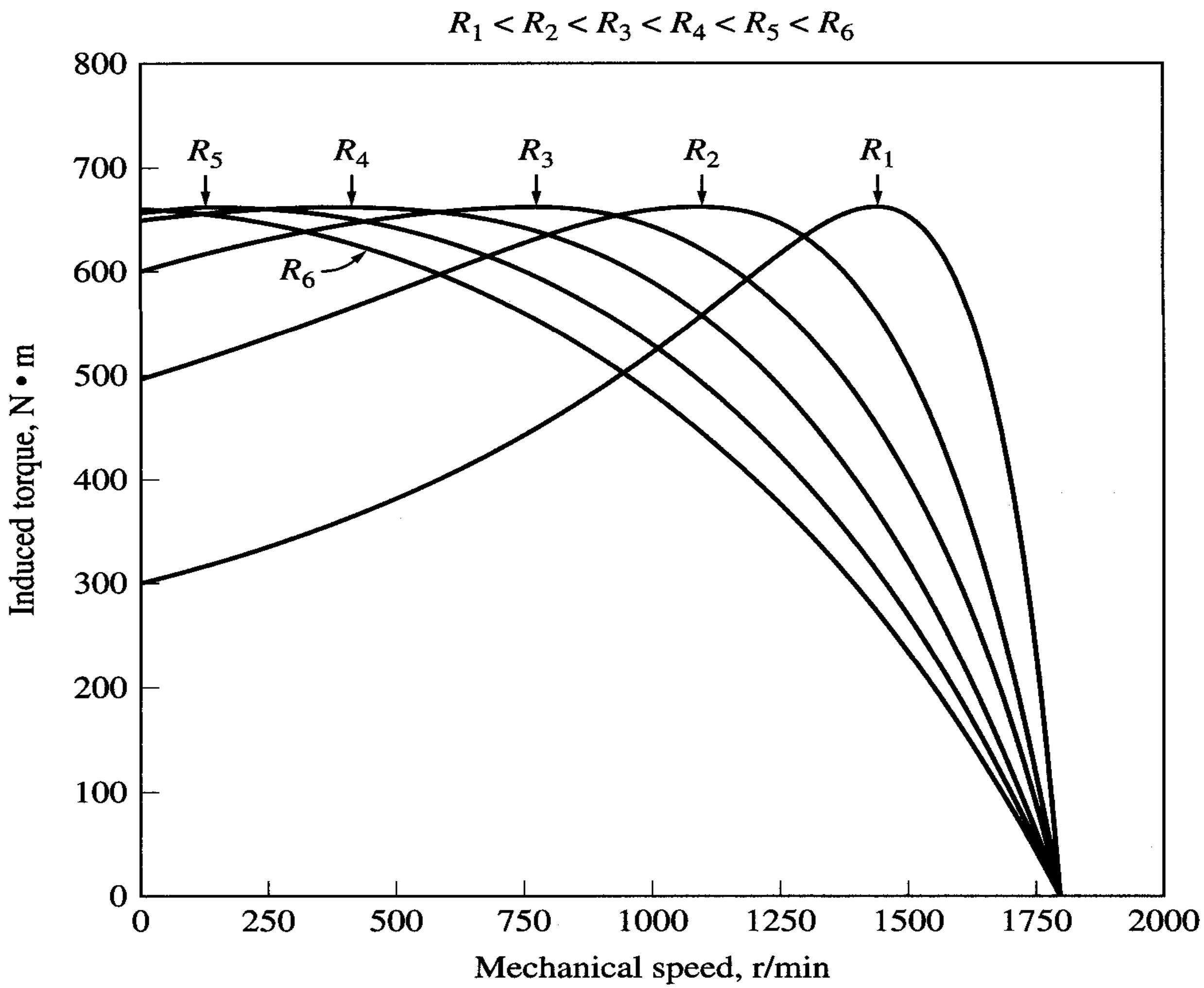
The

value of the maximum torque remains unaffected

but

the speed at which it occurs can be controlled.

Maximum torque



Effect of rotor resistance on torque-speed characteristic

Example

A two-pole, 50-Hz induction motor supplies 15kW to a load at a speed of 2950 rpm.

1. What is the motor's slip?
2. What is the induced torque in the motor in N.m under these conditions?
3. What will be the operating speed of the motor if its torque is doubled?
4. How much power will be supplied by the motor when the torque is doubled?

Solution

1. $$n_{sync} = \frac{120 f_e}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$

$$s = \frac{n_{sync} - n_m}{n_{sync}} = \frac{3000 - 2950}{3000} = 0.0167 \text{ or } 1.67\%$$

$\because$ no P_{f+W} given

2. $\therefore$ assume $P_{conv} = P_{load}$ and $\tau_{ind} = \tau_{load}$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{15 \times 10^3}{2950 \times \frac{2\pi}{60}} = 48.6 \text{ N.m}$$

Solution

3. In the low-slip region, the torque-speed curve is linear and the induced torque is direct proportional to slip. So, if the torque is doubled the new slip will be 3.33% and the motor speed will be

$$n_m = (1-s)n_{sync} = (1-0.0333)\times 3000 = 2900 \text{ rpm}$$

$$P_{conv} = \tau_{ind}\omega_m$$

$$= (2\times 48.6)\times(2900\times\frac{2\pi}{60}) = 29.5 \text{ kW}$$

4.

Example

A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit

R_1= 0.641Ω R_2= 0.332Ω

X_1= 1.106 Ω X_2= 0.464 Ω X_M= 26.3 Ω

1. What is the maximum torque of this motor? At what speed and slip does it occur?
2. What is the starting torque of this motor?
3. If the rotor resistance is doubled, what is the speed at which the maximum torque now occur? What is the new starting torque of the motor?

Solution

$$V_{TH} = V_{\phi} \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$= \frac{\frac{460}{\sqrt{3}} \times 26.3}{\sqrt{(0.641)^2 + (1.106 + 26.3)^2}} = 255.2 \text{ V}$$

$$R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2$$

$$\approx (0.641) \left(\frac{26.3}{1.106 + 26.3} \right)^2 = 0.590\Omega$$

$$X_{TH} \approx X_1 = 1.106\Omega$$

Solution

1. $$s_{T_{\max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$= \frac{0.332}{\sqrt{(0.590)^2 + (1.106 + 0.464)^2}} = 0.198$$

The corresponding speed is

$$n_m = (1 - s)n_{sync} = (1 - 0.198) \times 1800 = 1444 \text{ rpm}$$

Solution

The torque at this speed is

$$\tau_{\max} = \frac{1}{2\omega_s}\left(\frac{3V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}\right)$$

$$= \frac{3\times(255.2)^2}{2\times(1800\times\frac{2\pi}{60})[0.590+\sqrt{(0.590)^2+(1.106+0.464)^2}\,]}$$

$$= 229 \text{ N.m}$$

Solution

2. The starting torque can be found from the torque eqn. by substituting $s = 1$

$$\tau_{start} = \tau_{ind}\Big|_{s=1} = \frac{1}{\omega_s} \frac{3V_{TH}^2\left(\dfrac{R_2}{s}\right)}{\left(R_{TH} + \dfrac{R_2}{s}\right)^2 + (X_{TH} + X_2)^2}\Bigg|_{s=1}$$

$$= \frac{3V_{TH}^2 R_2}{\omega_s[\left(R_{TH} + R_2\right)^2 + (X_{TH} + X_2)^2]}$$

$$= \frac{3\times(255.2)^2\times(0.332)}{1800\times\dfrac{2\pi}{60}\times[(0.590+0.332)^2+(1.106+0.464)^2]}$$

$$= 104 \text{ N.m}$$

Solution

3. If the rotor resistance is doubled, then the slip at maximum torque doubles too

$$s_{T_{\max}} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}} = 0.396$$

The corresponding speed is

$$n_m = (1-s)n_{sync} = (1-0.396) \times 1800 = 1087 \text{ rpm}$$

The maximum torque is still

τ_{max} = 229 N.m

Solution

The starting torque is now

$$\tau_{start} = \frac{3\times(255.2)^2\times(0.664)}{1800\times\dfrac{2\pi}{60}\times[(0.590+0.664)^2+(1.106+0.464)^2]}$$

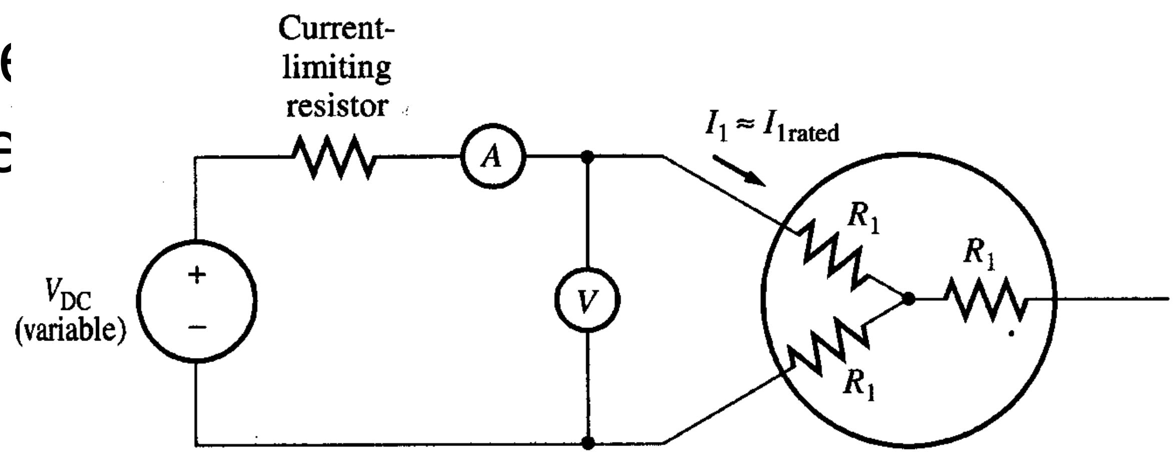
$$= 170 \text{ N.m}$$

Determination of motor parameters

- Due to the similarity between the induction motor equivalent circuit and the transformer equivalent circuit, same tests are used to determine the values of the motor parameters.
  - DC test: determine the stator resistance R_1
  - No-load test: determine the rotational losses and magnetization current (similar to no-load test in Transformers).
  - Locked-rotor test: determine the rotor and stator impedances (similar to short-circuit test in

DC test

- The purpose of the DC test is to determine R_1. A variable DC voltage source is connected between two stator terminals.
- The DC source is adjusted to provide approximately rated stator current, and the resistance between the two stator leads is de neter re



DC test

– then

$$R_{DC} = \frac{V_{DC}}{I_{DC}}$$

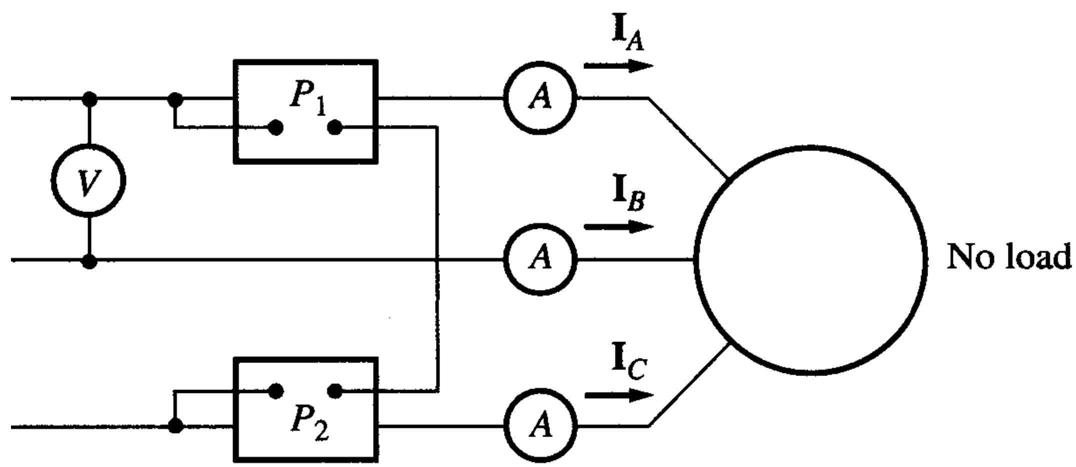
– If the stator is Y-connected, the per phase stator resistance is

$$R_1 = \frac{R_{DC}}{2}$$

– If the stator is delta-connected, the per phase stator resistance is

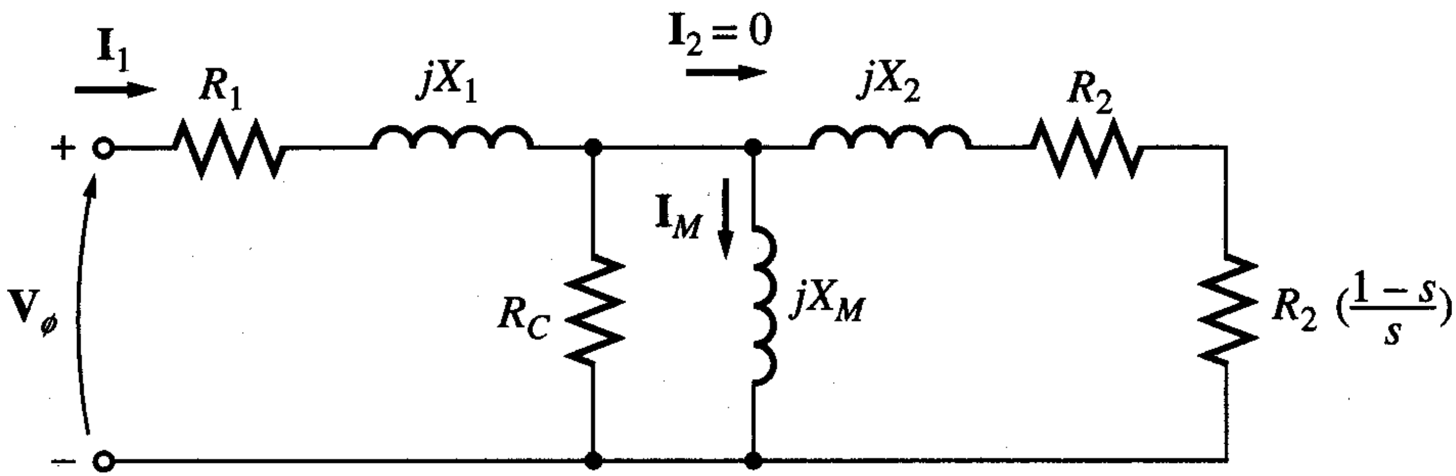
$$R_1 = \frac{3}{2} R_{DC}$$

No-load test



1. The motor is allowed to spin freely
2. The only load on the motor is the friction and windage losses, so all P_{conv} is consumed by mechanical losses
3. The slip is very small

No-load test

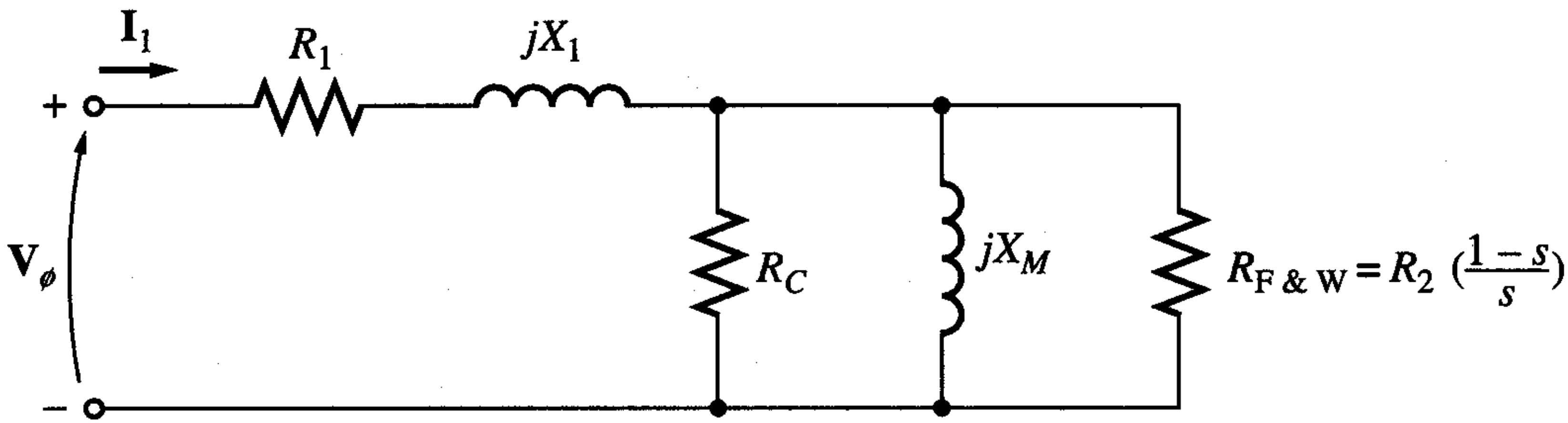


4. At this small slip

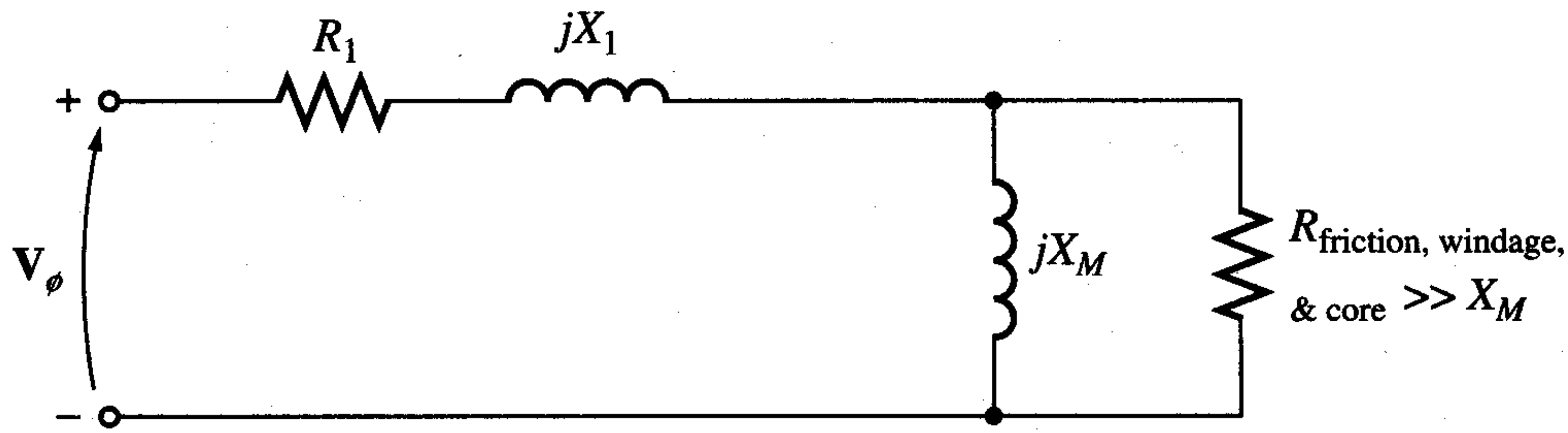
$$\frac{R_2(1-s)}{s} \gg R_2 \qquad \& \qquad \frac{\mathrm{R}_2(1-s)}{s} \gg X_2$$

The equivalent circuit reduces to…

No-load test



5. Combining R_c & R_{F+W} we get……



No-load test

6. At the no-load conditions, the input power measured by meters must equal the losses in the motor.
7. The P_{RCL} is negligible because I_2 is extremely small because $R_2(1\text{-}s)/s$ is very large.
8. The input power equals

$$\begin{aligned} P_{in} &= P_{SCL} + P_{core} + P_{F\&W} \\ &= 3I_1^2 R_1 + P_{rot} \end{aligned}$$

$$P_{rot} = P_{core} + P_{F\&W}$$

Where

No-load test

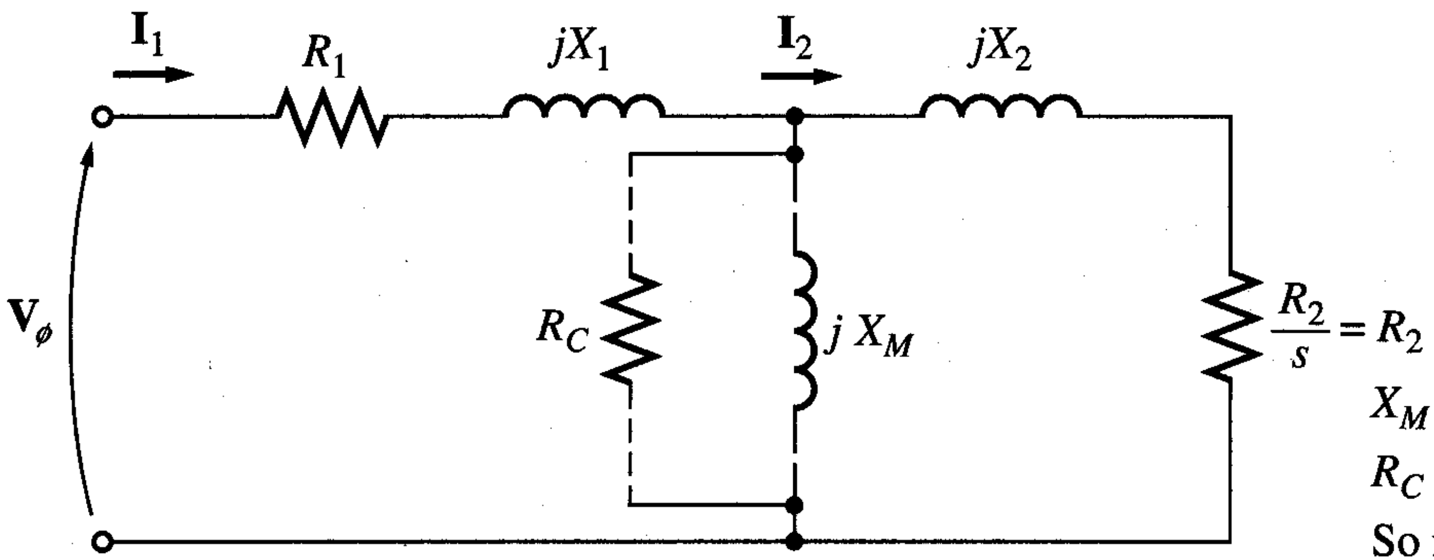
9. The equivalent input impedance is thus approximately

$$\left\| Z_{eq} \right\| = \frac{V_\phi}{I_{1,nl}} \approx X_1 + X_M$$

If X_1 can be found, in some other fashion, the magnetizing impedance X_M will be known

Blocked-rotor test

- In this test, the rotor is locked or blocked so that it cannot move, a voltage is applied to the motor, and the resulting voltage, current and power are measured.



$X_M >> |R_2 + jX_2|$

$R_C >> |R_2 + jX_2|$

So neglect R_C and X_M

Blocked-rotor test

- The AC voltage applied to the stator is adjusted so that the current flow is approximately full-load value.
- The locked-rotor power factor can be found as

$$PF = \cos\theta = \frac{P_{in}}{\sqrt{3}V_l I_l}$$

- The magnitude of the total impedance

$$|Z_{LR}| = \frac{V_\phi}{I}$$

Blocked-rotor test

$$\left|Z_{LR}\right| = R_{LR} + jX'_{LR}$$
$$= \left|Z_{LR}\right|\cos\theta + j\left|Z_{LR}\right|\sin\theta$$

$$R_{LR} = R_1 + R_2$$
$$X'_{LR} = X'_1 + X'_2$$

Where X'_1 and X'_2 are the stator and rotor reactances at the test frequency respectively

$$R_2 = R_{LR} - R_1$$

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

Blocked-rotor test

	X_1 and X_2 as function of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

Midterm Exam No.2

Example

The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected IM having a rated current of 28 A.

DC Test:

V_{DC} = 13.6 V $\quad$ I_{DC} = 28.0 A

No-load Test:

V_l = 208 V $\quad$ f = 60 Hz

I = 8.17 A $\quad$ P_{in} = 420 W

Locked-rotor Test:

V_l = 25 V $\quad$ f = 15 Hz

I = 27.9 A $\quad$ P_{in} = 920 W

(a) Sketch the per-phase equivalent circuit of this motor.

(b) Find the slip at pull-out torque, and find the value of the pull-out torque.

UNIT-IV

SYNCHRONOUS AND SPECIAL MACHINES

SYNCHRONOUS GENERATORS
Summary

1. **Synchronous Generator Construction**
2. **Speed of Rotation of a Synchronous Generator**
3. **Internal Voltage of a Synchronous Generator**
4. **Equivalent Circuit of a Synchronous Generator**
5. **Phasor Diagram of a Synchronous Generator Eq. cct.**
6. **Power and Torque in Synchronous Generator**
7. **Measuring Synchronous Generator Model Parameters**

SYNCHRONOUS GENERATOR CONSTRUCTION

- syn. gen. used to convert mechanical energy to ac electric energy: generators in power plants
- steady state operation of synchronous generators discussed here
- generator construction

  - in synchronous generator, rotor winding energized by dc source to develop rotor magnetic field

  - rotor is turned by a prime mover, producing a rotating magnetic field which induce 3 phase voltages in stator windings

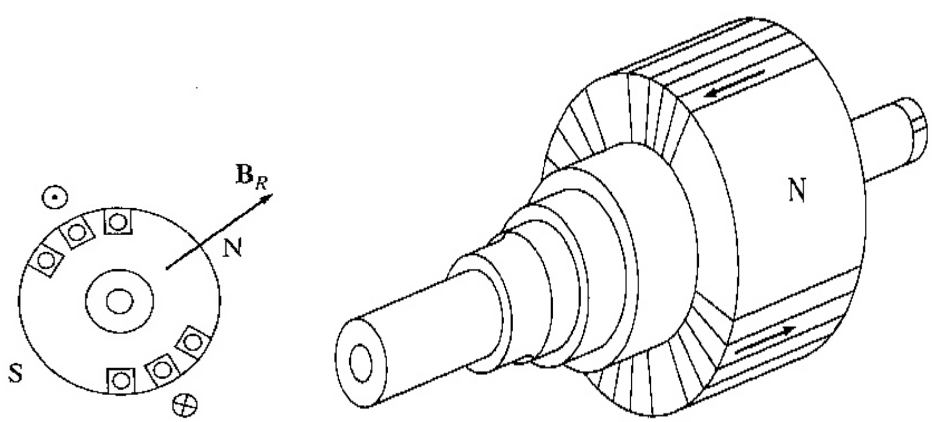
  In general rotor carry the "field windings" , while "armature windings" (or "stator windings") carry the main voltages of machine
- therefore:

- rotor windings $\equiv$ field windings
- stator windings $\equiv$ armature windings

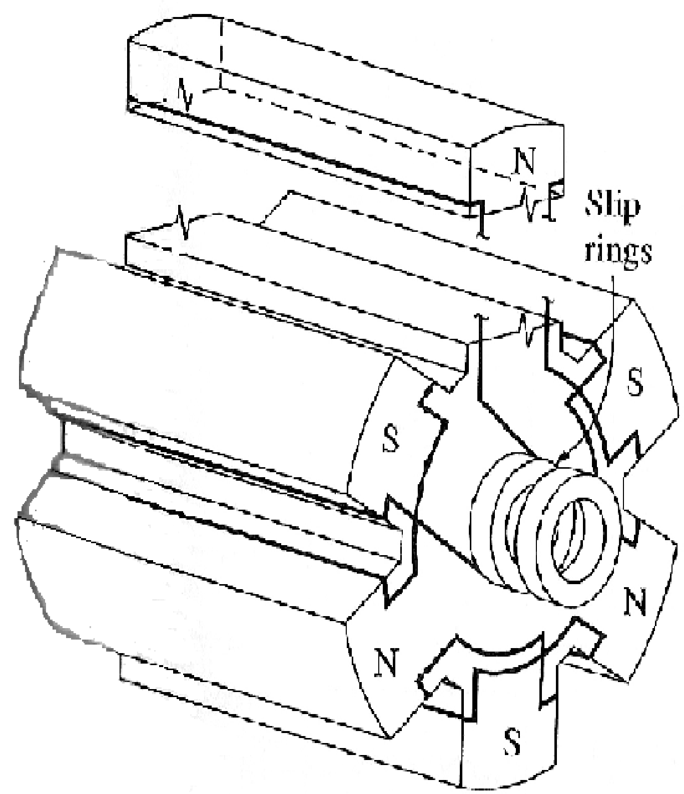
SYNCHRONOUS GENERATOR CONSTRUCTION

- Rotor of synchronous machine can be

Nonsalient: 2 pole rotor   **Salient:** six-pole rotor

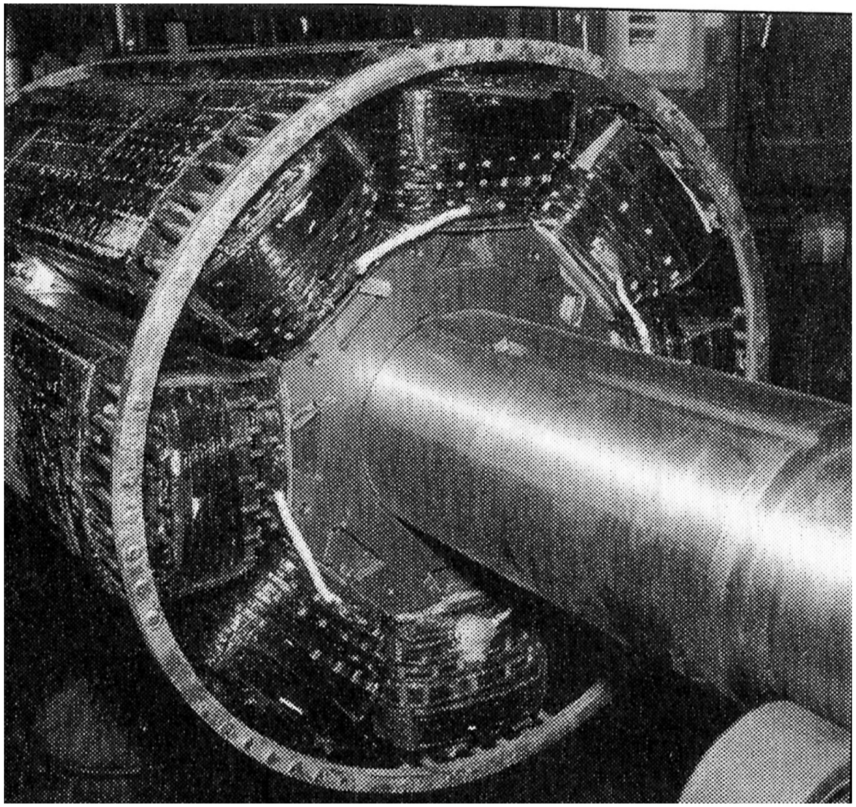


Side View



SYNCHRONOUS GENERATOR CONSTRUCTION

- Photograph of a salient 8-pole synchronous machine rotor



SYNCHRONOUS GENERATOR CONSTRUCTION

- **Rotor** experience varying magnetic fields, therefore is constructed of thin laminations to reduce eddy current losses
- To supply the rotor winding while it is rotating, special arrangement employed to connect its terminal to dc supply

1. supply dc power from an external dc source to rotor by means of slip rings
2. supply dc power from a special dc power source mounted on shaft of rotor

SYNCHRONOUS GENERATOR CONSTRUCTION

- **SLIP RINGS**: are metal rings encircling shaft and are insulated from it

  - one end of rotor winding is connected to each of the 2 slip rings

  - and a stationary brush mounted on the machine casing ride on each slip ring
- **Brush:** a block of graphite like carbon compound that conducts and has low friction
- same dc voltage is applied to field winding during rotation

SYNCHRONOUS GENERATOR CONSTRUCTION

- Problems associated with slip rings and brushes:

  1- increase the required maintenance (brushes should be examined for wear regularly)

  2- brush voltage drop results in significant power losses if field current is high

- Despite of above problems, SLIP RINGS & BRUSHES used for smaller synchronous machines since is cost-effective

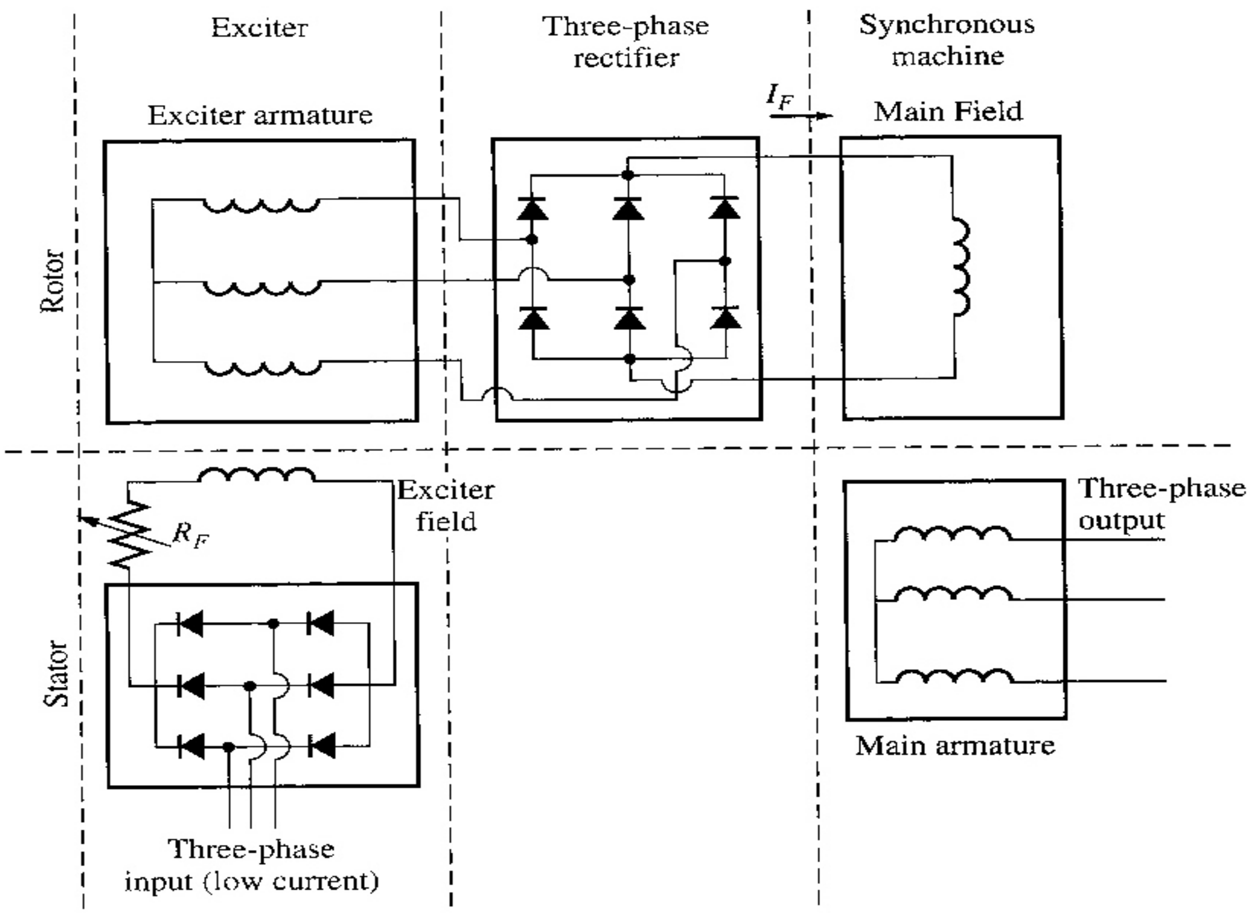
SYNCHRONOUS GENERATOR CONSTRUCTION

- on larger generator & motors, brushless exciters are used
- **Brushless Exciter:** is a smaller ac generator with its field circuit mounted on stator & its armature circuit mounted on rotor shaft

  - 3 phase output of exciter generator rectified by a 3 phase rectifier mounted also on shaft
- By controlling small dc field current of exciter generator, it is possible to fed (and also adjust) field current of main machine without slip rings and brushes

SYNCHRONOUS GENERATOR CONSTRUCTION

- Schematic arrangement of a brushless exciter



SYNCHRONOUS GENERATOR CONSTRUCTION

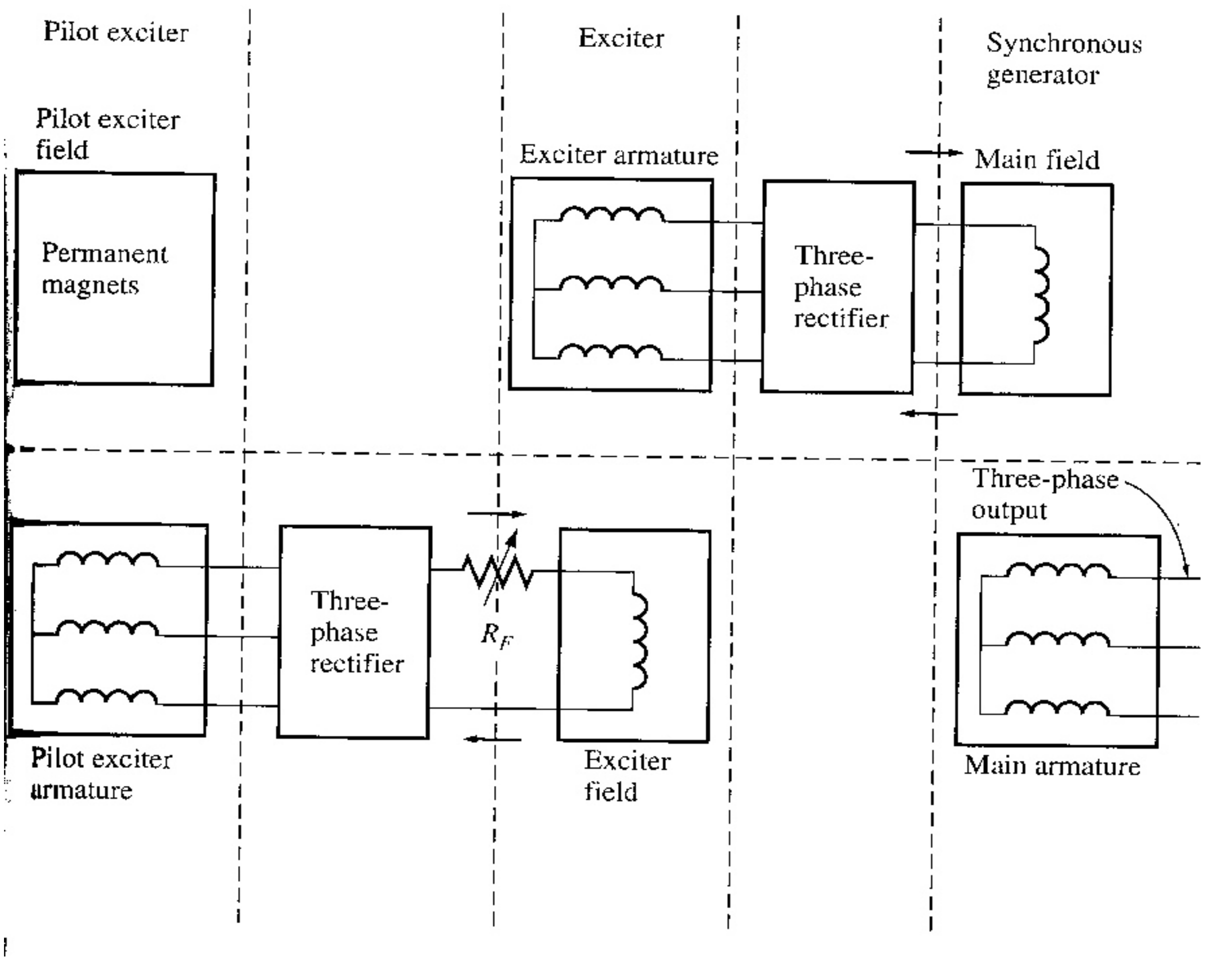
- Photograph of a synchronous machine with brushless exciter

SYNCHRONOUS GENERATOR CONSTRUCTION

- A small pilot exciter often included in system to have the excitation of generator independent of any external power sources
- A pilot exciter is a small ac generator with permanent magnets mounted on rotor shaft & a 3 phase winding on stator
- It produces power for field circuit of exciter, which in turn controls the field circuit of main machine
- With pilot exciter on shaft of generator, no external electric power is required to run generator
- Many Syn. Gen.s with brushless exciters also have slip rings and brushes, as an auxiliary source of dc field in emergencies

SYNCHRONOUS GENERATOR CONSTRUCTION

- Brushless exciter including a pilot exciter



SYNCHRONOUS GENERATOR

Speed of rotation of synchronous generator

- synchronous generators are *synchronous, during their operation*

means: electrical frequency is synchronized with mechanical speed of rotor

- Relation between electrical frequency of stator and mechanical speed of rotor as shown before: $f_e = n_m p / 120$

f_e : electrical frequency in Hz

n_m: speed of rotor in r/min

p: number of poles

SYNCHRONOUS GENERATOR

Speed of rotation of synchronous generator

- Electric power generated at 50 or 60 Hz, so rotor must turn at fixed speed depending on number of poles on machine
- To generate 60 Hz in 2 pole machine, rotor must turn at 3600 r/min, and to generate 50 Hz in 4 pole machine, rotor must turn at 1500 r/min
- **INTERNAL GENERATED VOLTAGE OF A SYNCHRONOUS GENERATOR**
- magnitude of induced voltage in one phase determined in last section: $E_A = \sqrt{2}\,\pi\, N_C\, \phi\, f$

SYNCHRONOUS GENERATOR
INTERNAL GENERATED VOLTAGE

- Induced voltage depends on flux ϕ, frequency or speed of rotation f, & machine's construction
- Last equation can be rewritten as:

$E_A = K \phi \omega$

$K = N_C/\sqrt{2} \qquad (\text{if } \omega = \omega_e)$

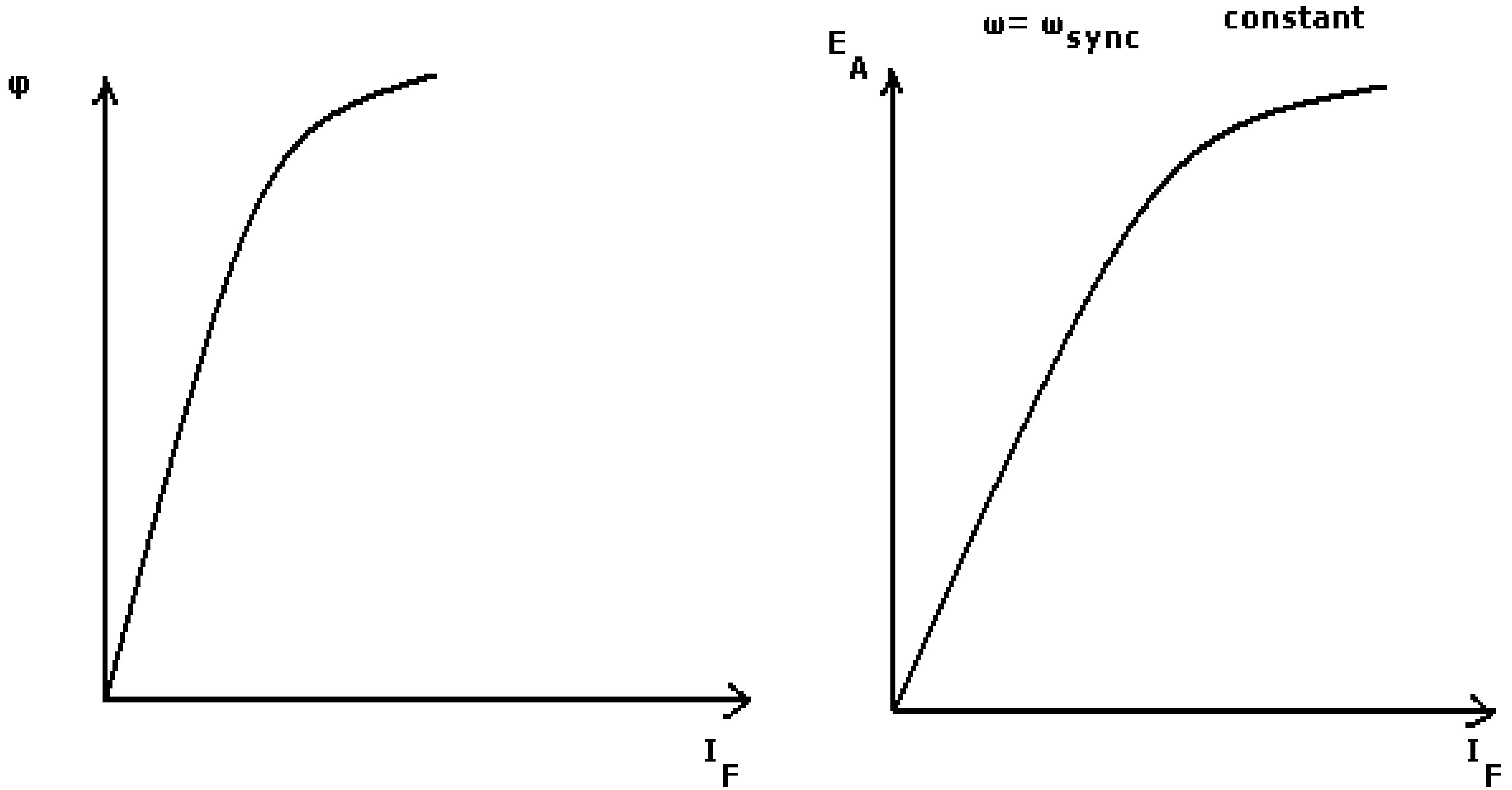
$K = N_C\, p/\sqrt{2} \qquad (\text{if } \omega = \omega_m)$

- Note: E_A proportional to flux & speed, while flux depend on current in rotor winding I_F , therefore E_A is related to I_F & its plot named: magnetization curve, or O/C characteristic

SYNCHRONOUS GENERATOR

INTERNAL GENERATED VOLTAGE

- Plots of flux vs I_F and magnetization curve



SYNCHRONOUS GENERATOR EQUIVALENT CIRCUIT

- To develop a relation for V_ϕ as terminal voltage of generator which is different from internal voltage E_A equivalent circuit is needed
- Reasons for V_ϕ to be different from E_A

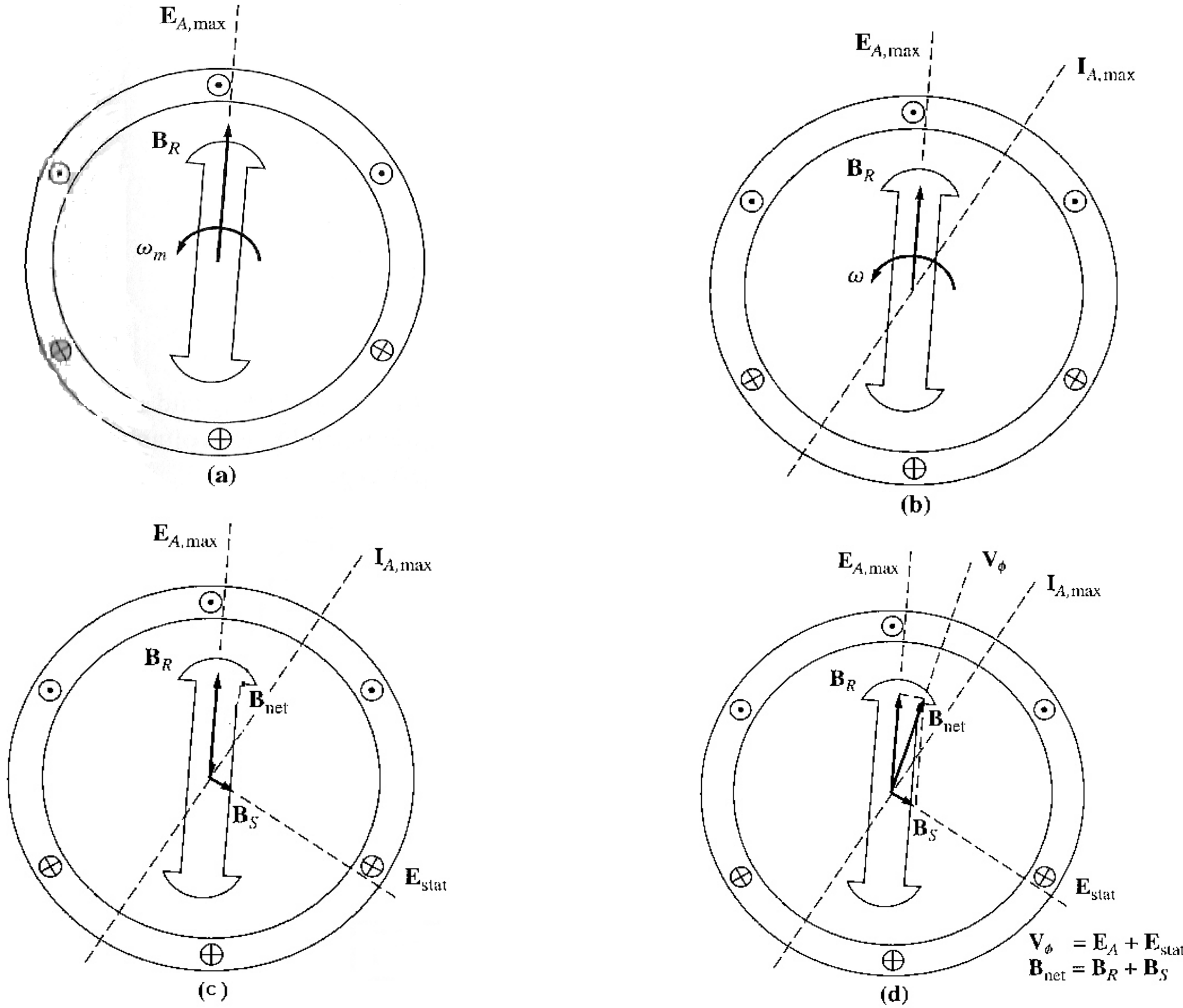
1- distortion of air-gap magnetic field magnetic field due to current flowing in stator, called *armature reaction*

2- self-inductance of armature coils

3- resistance of armature coils

4- effect of salient-pole rotor shapes (ignored as machines have cylindrical rotors)

SYNCHRONOUS GENERATOR EQ. CCT. (ARM. REAC.)



SYNCHRONOUS GENERATOR EQ. CCT. (ARM. REAC)...

- Last figure shows a 2 pole rotor spinning inside a 3 phase stator, without load
- Rotor magnetic field B_R develop a voltage E_A as discussed in last chapter voltage is positive out of conductors, at top, and negative into the conductors at bottom of figure
- When there is no load on generator, armature current zero, $E_A = V_\phi$
- If generator be connected to a lagging load, peak current occur at an angle behind peak voltage as in fig (b)

SYNCHRONOUS GENERATOR EQ. CCT. (ARM. REAC)…

- Current flowing in stator windings produces its magnetic field
- Stator magnetic field named B_S & its direction found by R.H.R. as shown in fig(c) this B_S produces another voltage in stator, named E_{stat} and shown in figure
- Having these 2 voltage components in stator windings, total voltage in one phase is sum of E_A and E_{stat} :

$V_\phi = E_A + E_{stat}$ and $B_{net} = B_R + B_S$

angle of B_{net} coincide with angle of V_ϕ shown in fig (d)

SYNCHRONOUS GENERATOR EQ. CCT. (ARM. REAC)...

- To model effect of armature reaction, note:

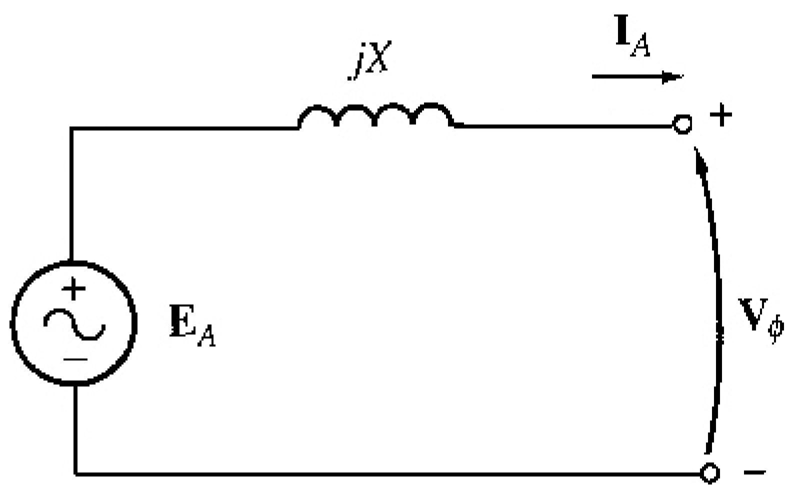
  1- E_{stat} lies at an angle of 90° behind plane of maximum current I_A

  2- E_{stat} directly proportional to I_A and X is constant of proportionality

➔ $E_{stat} = -j\,X\,I_A$

➔ voltage in one phase $V_\phi = E_A - j\,X\,I_A$

- Following eq. cct. can be developed



SYNCHRONOUS GENERATOR EQ. CCT. (ARM. REAC)...

- Armature reaction voltage can be modeled as an inductor in series with internal induced voltage
- In addition to armature reaction, stator coils have a self-inductance and a resistance
- stator self-inductance named L_A (its reactance X_A) and stator resistance is R_A :

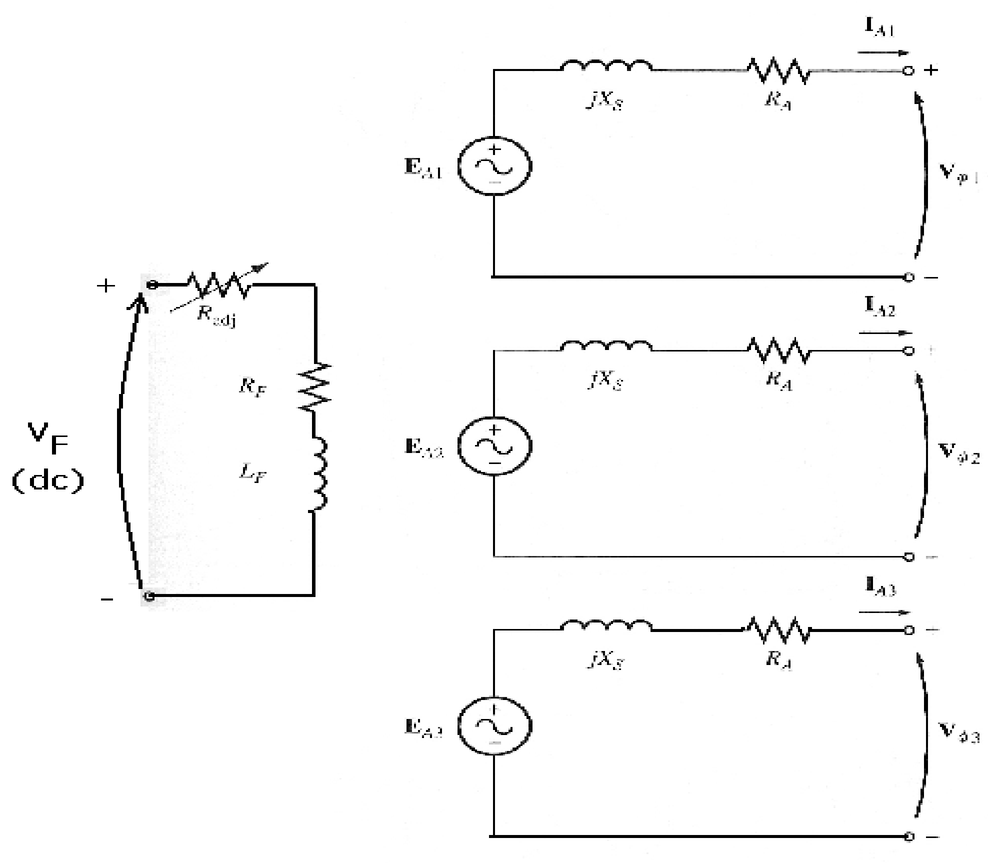
$$V_\phi = E_A - jXI_A - jX_A I_A - R_A I_A$$

- Armature reaction & self-inductance in machine both represented by reactance, normally they are combined to a single reactance as : $X_S = X + X_A$

$$V_\phi = E_A - jX_S I_A - R_A I_A$$

SYNCHRONOUS GENERATOR EQ. CCT. (ARM. REAC)...

- equivalent circuit of a 3 phase synchronous generator can be shown as follows:

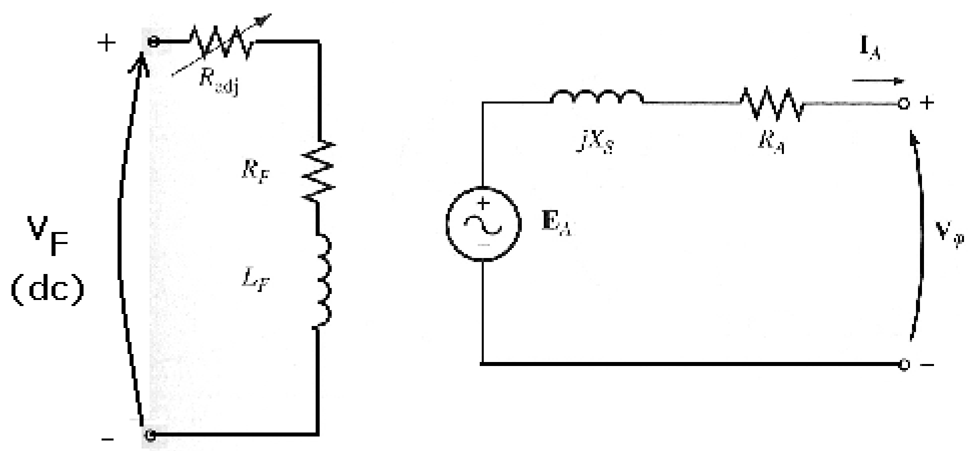


SYNCHRONOUS GENERATOR EQ. CCT. ...

- Figure shows a dc source, supplying rotor winding, modeled by coil inductance & resistance in series with an adjustable resistor R_{adj} that controls current
- Rest of equivalent circuit consists of model for each phase
- the voltages and currents of each phase are 120° apart with same magnitude
- Three phases can be connected in Y or Δ
- If connected in Y : $V_T = \sqrt{3}\, V_\phi$
- If connected in Δ: $V_T = V_\phi$

SYNCHRONOUS GENERATOR EQ. CCT…

- The per phase equivalent circuit is shown below



- can be employed when loads of 3 phase are balanced

Introduction

- Although all electric machines have the same basic principle of operation, special-purpose machines have some features that distinguish them from conventional machines.

- It is not our intention to discuss all kinds of special-purpose machines in one chapter; rather, an attempt is made to introduce the basic operating principles of some special-purpose machines that are being used extensively in home, recreational, and industrial applications.

- With the proliferation of power electronic circuits and digital control systems, precise speed and position control can be achieved in conjunction with special-purpose electric machines such as permanent-magnet (PM) motors, step motors, switched-reluctance motors, brushless direct-current (dc) motors, hysteresis motors, and linear motors.

Introduction

- Some of these devices find applications in computer peripheral equipment or in process-control systems whereas others can be used in devices such as home appliances.
- For example, step motors are employed extensively in computers where precise positioning is required, as in the case of a magnetic head for a disk drive.
- For applications that demand constant-speed drives, brushless dc motors offer excellent characteristics.
- Switched-reluctance motors, on the other hand, find applications where we traditionally use dc or induction motors.
- In the following sections we discuss the construction, operating principles, and characteristics of each of the above-mentioned special-purpose electric machines.

Permanent-Magnet Motors

- The development of new permanent-magnet materials has made PM motors a viable substitute for a shunt (dc) motor.
- In a PM motor the poles are made of permanent magnets, as shown in Figure 12.1.
- Although dc motors up to 75 hp have been designed with permanent magnets, the major application of permanent magnets is confined to fractional-horsepower motors for economic reasons.

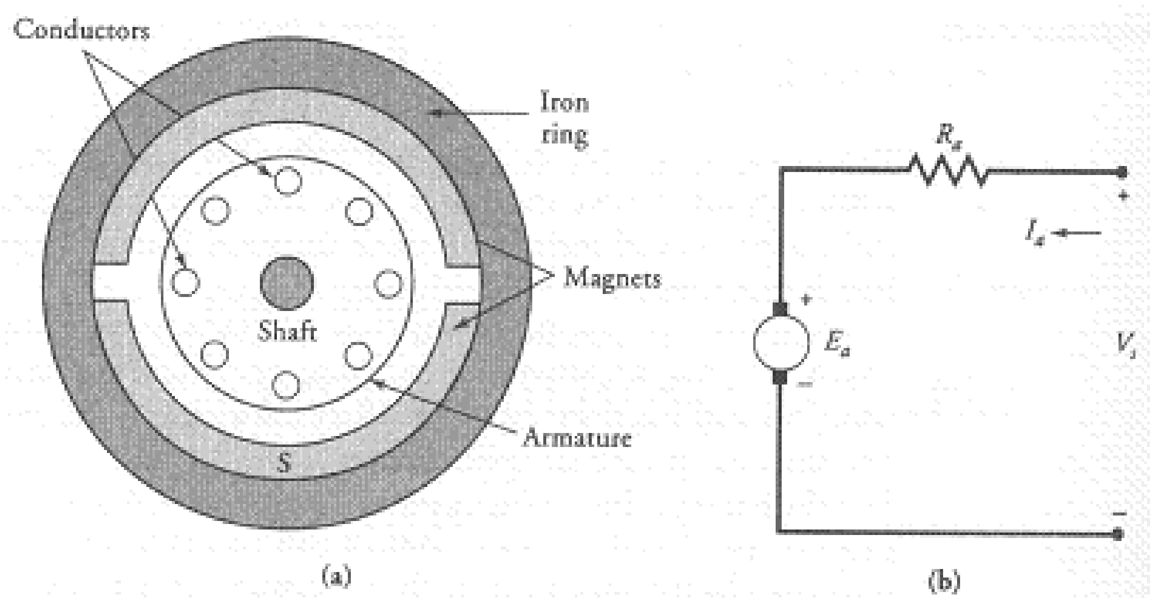


Figure 12.1 (a) Cross-sectional view of a PM motor; (b) equivalent circuit.

Permanent-Magnet Motors

- In a conventional dc motor with a wound-field circuit, flux per pole depends on the current through the field winding and can be controlled.
- However, flux in a PM motor is essentially constant and depends on the point of operation.
- For the same power output, a PM motor has higher efficiency and requires less material than a wound dc motor of the same ratings.
- However, the design of a PM motor should be such that the effect of demagnetization due to armature reaction, which is maximum at standstill, is as small as economically possible.
- Since the flux in a PM motor is fixed, the speed- and current-torque characteristics are basically straight lines as shown in Fig. 12.2 in the next slide.

Permanent-Magnet Motors

$$\omega_m = \frac{V_s}{K_a\Phi_p} - \frac{R_a}{(K_a\Phi_p)^2} T_d \tag{12.1}$$

and

$$I_a = \frac{1}{K_a\Phi_p} T_d \tag{12.2}$$

where K_a, V_s, Φ_p, R_a, and T_d are the machine constant, supply voltage, flux per pole, resistance of the armature winding, and developed torque.

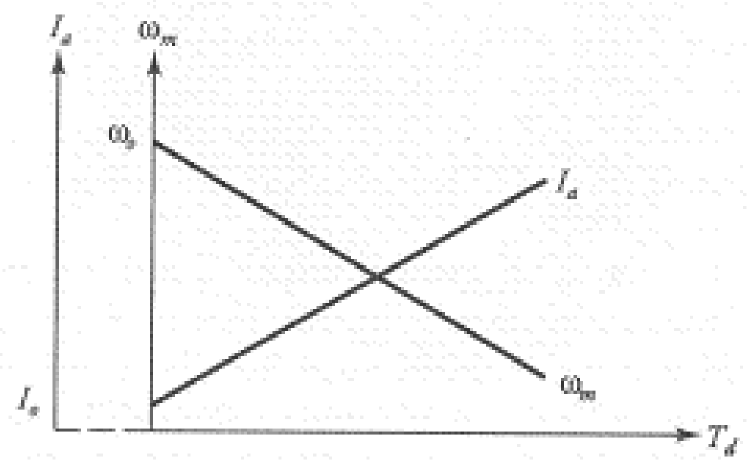


Figure 12.2 Speed- and current-torque characteristics of a PM motor.

Permanent-Magnet Motors

- The speed-torque characteristic of a PM motor can be controlled by changing either the supply voltage or the effective resistance of the armature circuit.
- The change in the supply voltage varies the no-load speed of the motor without affecting the slope of the characteristic.
- Thus for different supply voltages, a set of parallel speed-torque characteristics can be obtained, as illustrated in Figure 12.3.

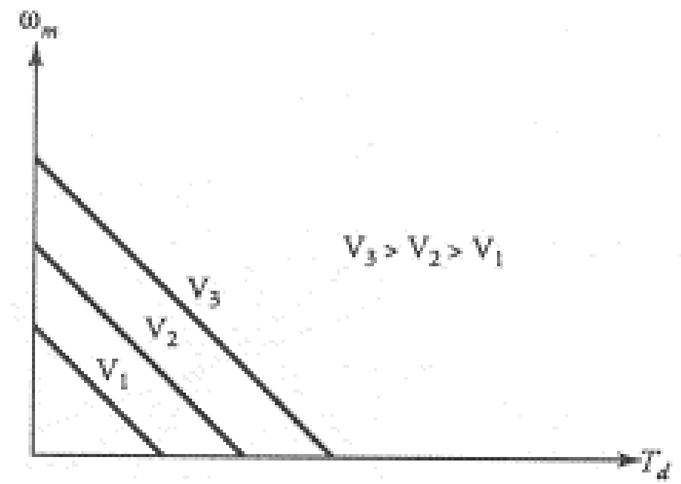


Figure 12.3 Operating characteristics for different supply voltages.

Permanent-Magnet Motors

- On the other hand, with the change in the effective resistance of the armature circuit, the slope of the curve is controlled and the no-load speed of the motor remains the same, as indicated in Figure 12.4.
- Using magnets with different flux densities and the same cross-sectional areas, or vice versa, there are almost infinite possibilities for designing a PM motor for a given operating condition, as shown in Figure 12.5.
- From the same figure we can also conclude that an increase in blocked-rotor torque can be achieved only at the expense of a lower no-load speed.

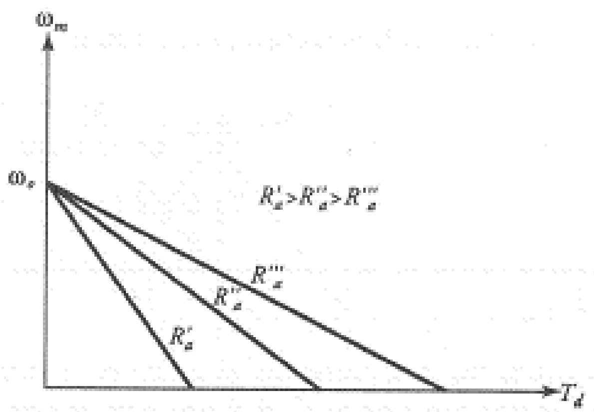


Figure 12.4 Operating characteristics for different resistances of armature circuit.

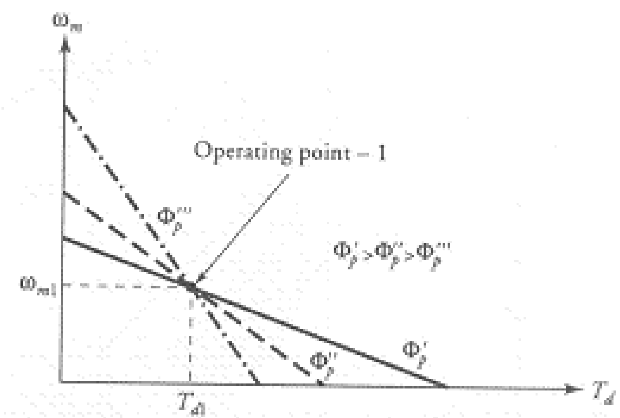


Figure 12.5 Operating characteristics for different fluxes in a PM motor.

Step Motors

- Step motors, also known as stepping or stepper motors, are essentially incremental motion devices.
- A step motor receives a rectangular pulse train and responds by rotating its shaft a certain number of degrees as dictated by the number of pulses in the pulse train.
- Usually the pulse train is controlled by means of a microcomputer or an electronic circuit.
- As a result, a step motor is very much compatible with digital electronic circuits and may form an interface between a microcomputer and a mechanical system.
- Since the motion in a step motor is generally governed by counting the number of pulses, no feedback loops and sensors are needed for its control.
- Therefore, step motors are suitable for position control in an open loop system.

Step Motors

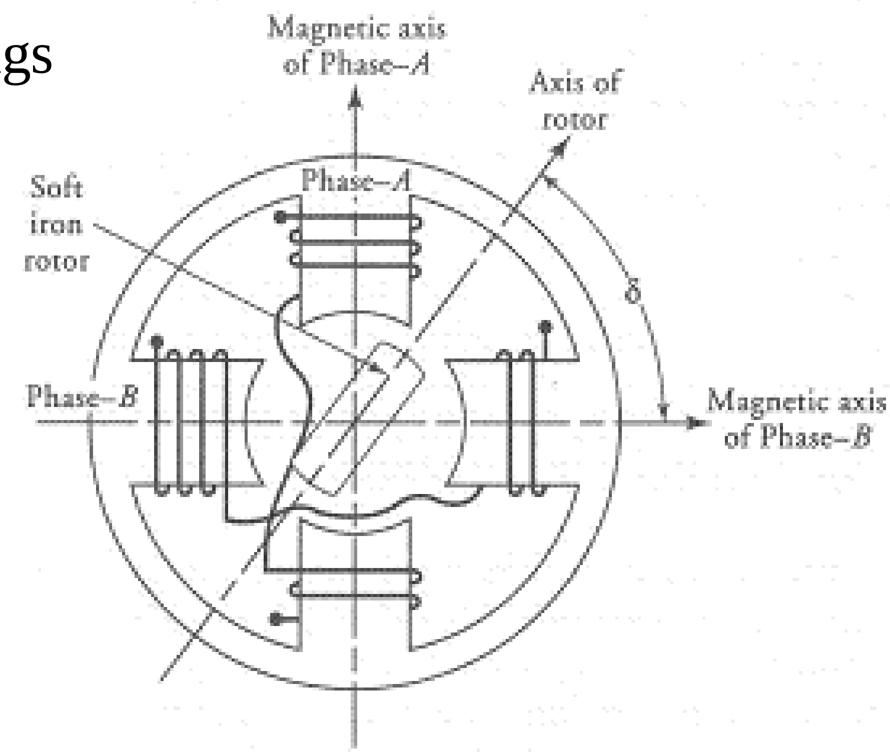
- They are relatively inexpensive and simple in construction and can be made to step in equal increments in either direction.
- Step motors are excellent candidates for such applications as printers, XY plotters, electric typewriters, control of floppy disk drives, robots, and numerical control of machine tools.
- Some of the drawbacks of step motors are that they do not offer the flexibility of adjusting the angle of advance, and their step response may be oscillatory in nature with a considerable overshoot.
- Step motors can be classified into three broad categories—variable-reluctance, permanent-magnet, and hybrid.
- Types:
  - Variable-reluctance step motors
  - Permanent-magnet step motors

Switched-Reluctance Motors

- In principle, a switched-reluctance motor operates like a variable-reluctance step motor discussed in the previous section.
- However, the operation differs mainly in the complicated control mechanism of the motor.
- In order to develop torque in the motor, the rotor position should be determined by sensors so that the excitation timing of the phase windings is precise.
- Although its construction is one of the simplest possible among electric machines, because of the complexities involved in the control and electric drive circuitry, switched-reluctance motors have not been able to find widespread applications for a long time.
- However, with the introduction of new power electronic and microelectronic switching circuits, the control and drive circuitry of a switched reluctance motor have become economically justifiable for many applications where traditionally dc or induction motors have been used.

Switched-Reluctance Motors

- A switched-reluctance motor has a wound stator but has no windings on its rotor, which is made of soft magnetic material as shown in Figure 12.17.
- The change in reluctance around the periphery of the stator forces the rotor poles to align with those of the stator.
- Consequently, torque develops in the motor and rotation takes place.
- The total flux linkages of phase-A in the following figure is $\lambda_a = L_a(\theta)\, i_a$ and of phase- B is $\lambda_b = L_b(\theta)\, i_b$ with the assumption that the magnetic materials are infinitely permeable.
- Since the magnetic axes of both windings are orthogonal, no mutual flux linkages are expected between them.

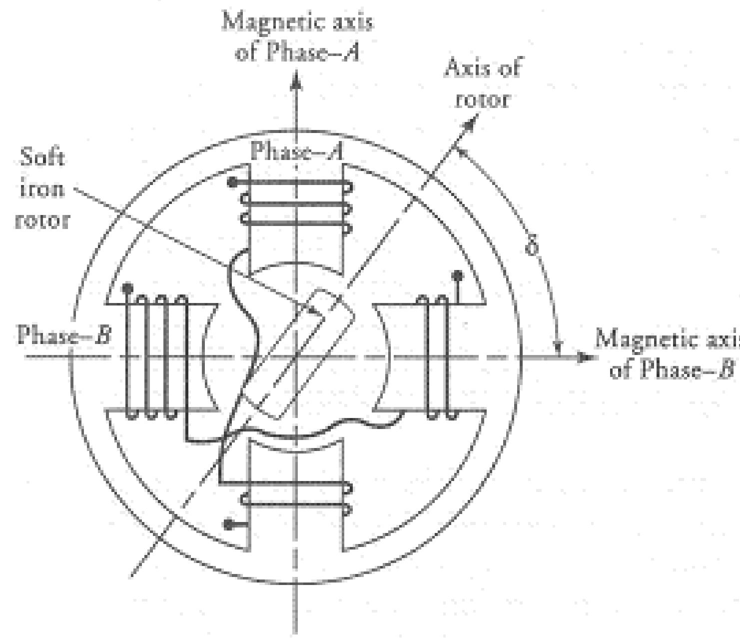


The co-energy in the motor is

$$W = \frac{1}{2} L_a(\theta) i_a^2 + \frac{1}{2} L_b(\theta) i_b^2$$

and the developed torque is

Switched-Reluctance Motors



The co-energy in the motor is

$$W = \frac{1}{2} L_a(\theta) i_a^2 + \frac{1}{2} L_b(\theta) i_b^2$$

and the developed torque is

$$T = \frac{dW}{d\theta}$$

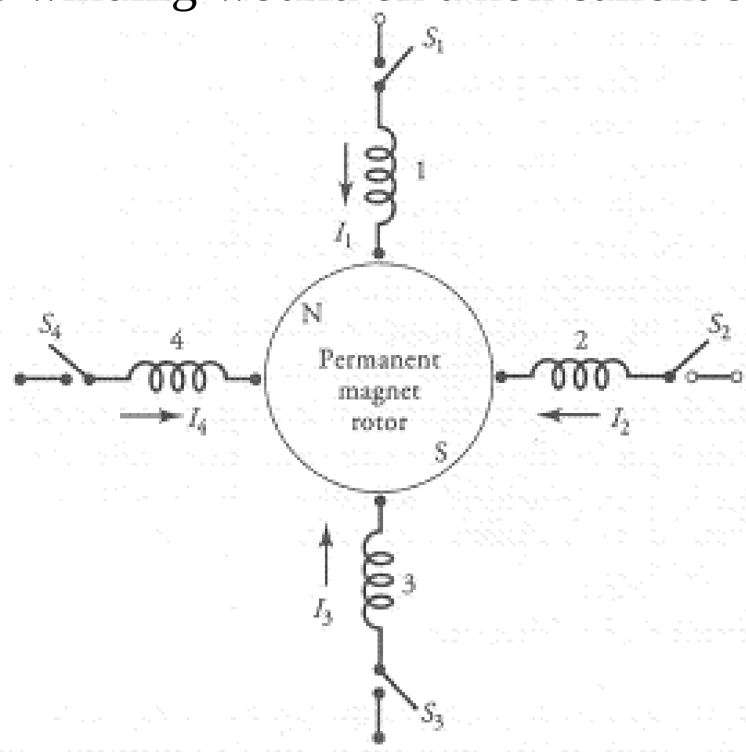
$$T = \frac{1}{2} i_a^2 \frac{dL_a}{d\theta} + \frac{1}{2} i_b^2 \frac{dL_b}{d\theta} \tag{12.5}$$

From Eq. (12.5) we can conclude that the developed torque in the motor is independent of the direction of the supply current because it is proportional to the square of the phase currents. However, the initial rotor position has a significant impact on the developed torque and rotation. Thus, a reliable rotor position sensor and a control circuitry are necessary to energize the motor at the proper instant to rotate it in the desired direction.

Brushless DC Motors

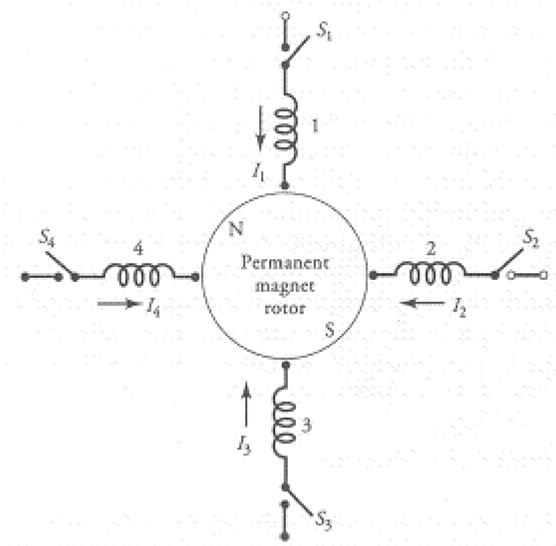
- DC motors find considerable applications where controlling a system is a primary objective.
- However, electric arcs produced by the mechanical commutator-brush arrangement are a major disadvantage and limit the operating speed and voltage.
- A motor that retains the characteristics of a dc motor but eliminates the commutator and the brushes is called a brushless dc motor.
- A brushless dc motor consists of a multiphase winding wound on a non-salient stator and a radially magnetized PM rotor.
- Figure 12.18 is a schematic diagram

  of a brushless dc motor.



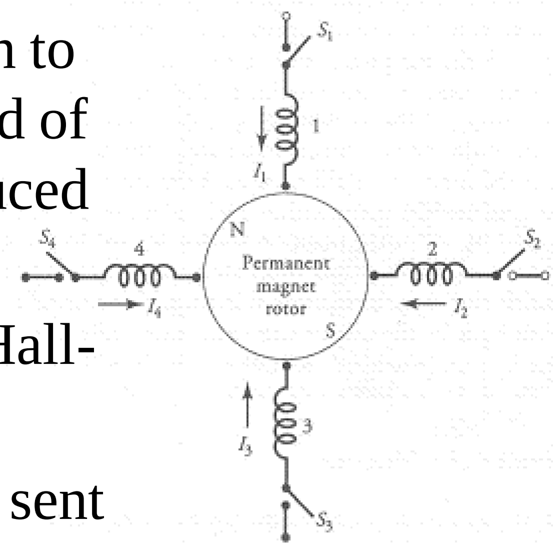
Brushless DC Motors

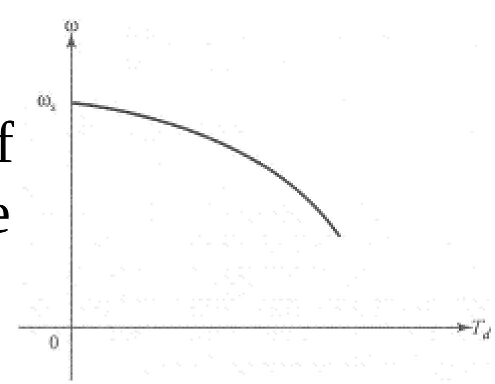
- Voltage is applied to individual phase windings through a sequential switching operation to achieve the necessary commutation to impart rotation.
- The switching is done electronically using power transistors or thyristors.
- For example, if winding 1 is energized, the PM rotor aligns with the magnetic field produced by winding 1.
- When winding 1 is switched off while winding 2 is turned on, the rotor is made to rotate to line up with the magnetic field of winding 2.



Brushless DC Motors

- As can be seen, the operation of a brushless dc motor is very similar to that of a PM step motor.
- The major difference is the timing of the switching operation, which is determined by the rotor position to provide the synchronism between the magnetic field of the permanent magnet and the magnetic field produced by the phase windings.
- The rotor position can be detected by using either Hall-effect or photoelectric devices.
- The signal generated by the rotor position sensor is sent to a logic circuit to make the decision for the switching, and then an appropriate signal triggers the power circuit to excite the respective phase winding.
- The control of the magnitude and the rate of switching of the phase currents essentially determine the speed-torque characteristic of a brushless dc motor, which is shown in Figure 12.19.



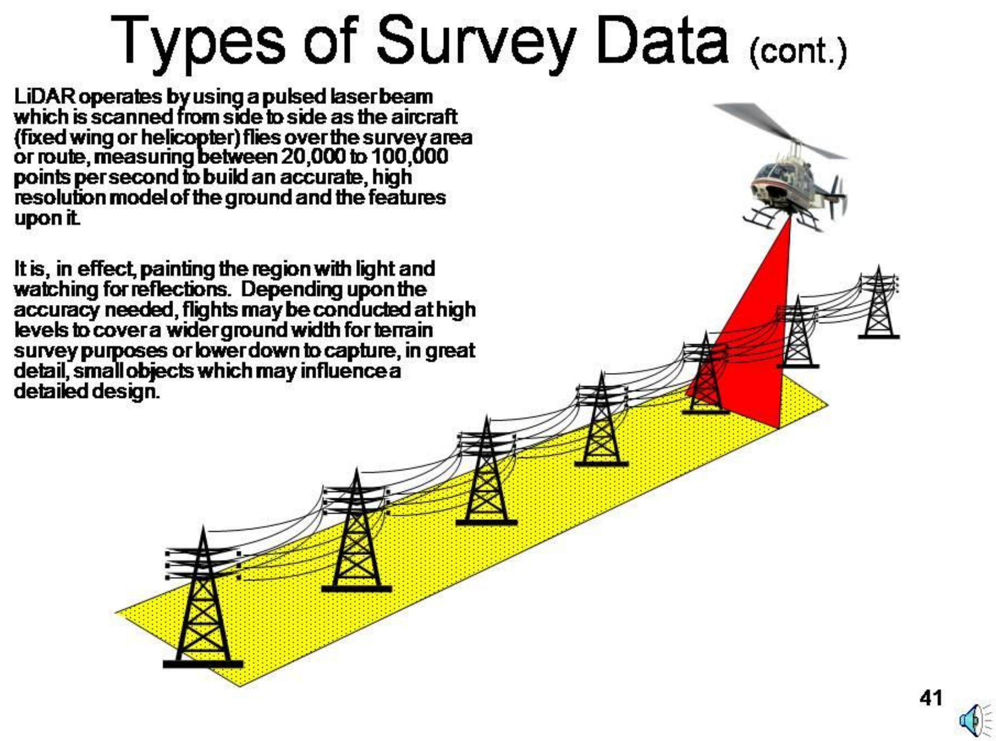


UNIT – V

Transmission & Distribution

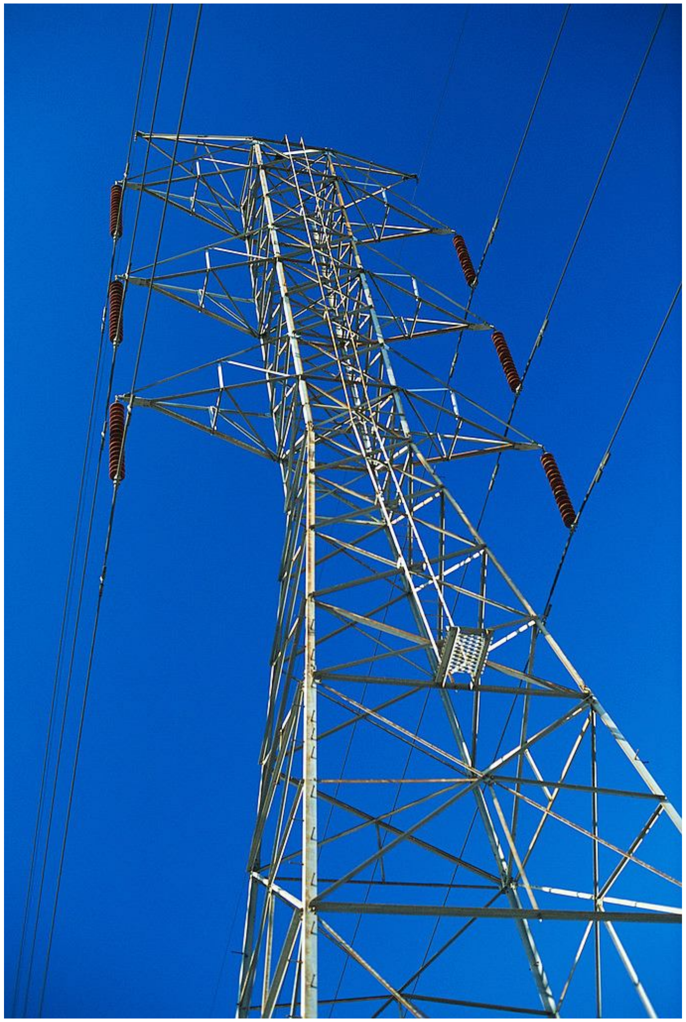
T&D Courses

Students learn from industry leaders the skills they need and how to use the latest tools in the T&D industry.



T&D Program

- Five course Certificate Program (15 credit hours)
- Content developed by industry experts **for** industry
- Industry practice and theory introduced in each course
- Experts provide practical details/calculation methods and GU Faculty oversee courses



T&D Program (cont.)

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Five utility related design courses:

- – Transmission Line Design – Introduction
- – Transmission Line Design – Advanced
- – Project Development & Construction Methods
- – Electrical Distribution System Design
- – Electric Grid Operations

TADP 540: Transmission Line Design-Introduction

Structures, conductors, insulation, grounding, survey techniques, terrain modeling, computer aided design, NESC code requirements.

Each major step in an overhead line design process will be analyzed and discussed using data from a recently constructed line. Advantages and disadvantages of some modern design tools will be established, and students will be provided the opportunity to use many of these same tools. The course is divided into four modules with each module emphasizing a key step in the line design process.

Load and Strength Reduction Factors (cont.)

Loads from suspended conductors are derived from appropriate weather district load cases.

Careful review of the construction grade requirements is needed.

Photo courtesy Alliant Energy

Overall, the course will provide civil, electrical, or mechanical engineering students an opportunity to think strategically about using standards-based methods in the design of overhead power lines over a given line route.

TADP 640:Transmission Line Design-Advanced

Advanced structures, foundation design, thermal conductor ratings, lightning protection, spotting algorithms, construction methods.

This course further develops the strategies that were learned in the Introduction course and introduces advanced concepts for designing Transmission lines.

TADP 544: Project Development and Construction Methods

Electrical grid design, system planning and project development, project proposals to management, project initiation, scheduling, cost management, resource management, permitting authority, land rights acquisition, overview of contracts, contractor selection, Gantt tracking.

This course will introduce students to typical transmission line projects. Students will study conductor types and uses, and learn strategies for developing and describing competing transmission projects. Given a specific transmission line project, the students will be able to develop a detailed project description in the form of a project plan.

TADP 541: Electrical Distribution System Design – Introduction

Network planning, protection/fusing, conductor sizing, transformer specification & connections, arrestors, reactive compensation, underground cabling, substation overview.

Students will learn the characteristics of distribution devices and how to select devices which contribute to the desired system performance. The course will cover the requirements of acceptable power quality and how to identify the different types of loads and their requirements for service.

TADP 543: Electrical Grid Operations

NERC/WECC reliability standards, control area operation, outage coordination planning, switch theory and devices, reactive load balancing, generation load balancing, economic dispatch, transmission marketing (OASIS), seasonal ratings.

An examination and study of the fundamental operating principles, guidelines and policies of the WECC (Western Electricity Coordinating Council) that promote reliable interconnected grid operation.

Future Courses

Substation Design

Relay Protection (Introduction and Advanced)

Environmental Aspects

Legal Aspects

Management and Leadership

Anticipate developing a Master's degree in **Engineering Design and Management**

Course Information

- Eight-week on-line courses – instead of 15 weeks
- Six contact hours/week – instead of three
- Each course split into four modules
- One industry expert will teach a two-week module

Target Students

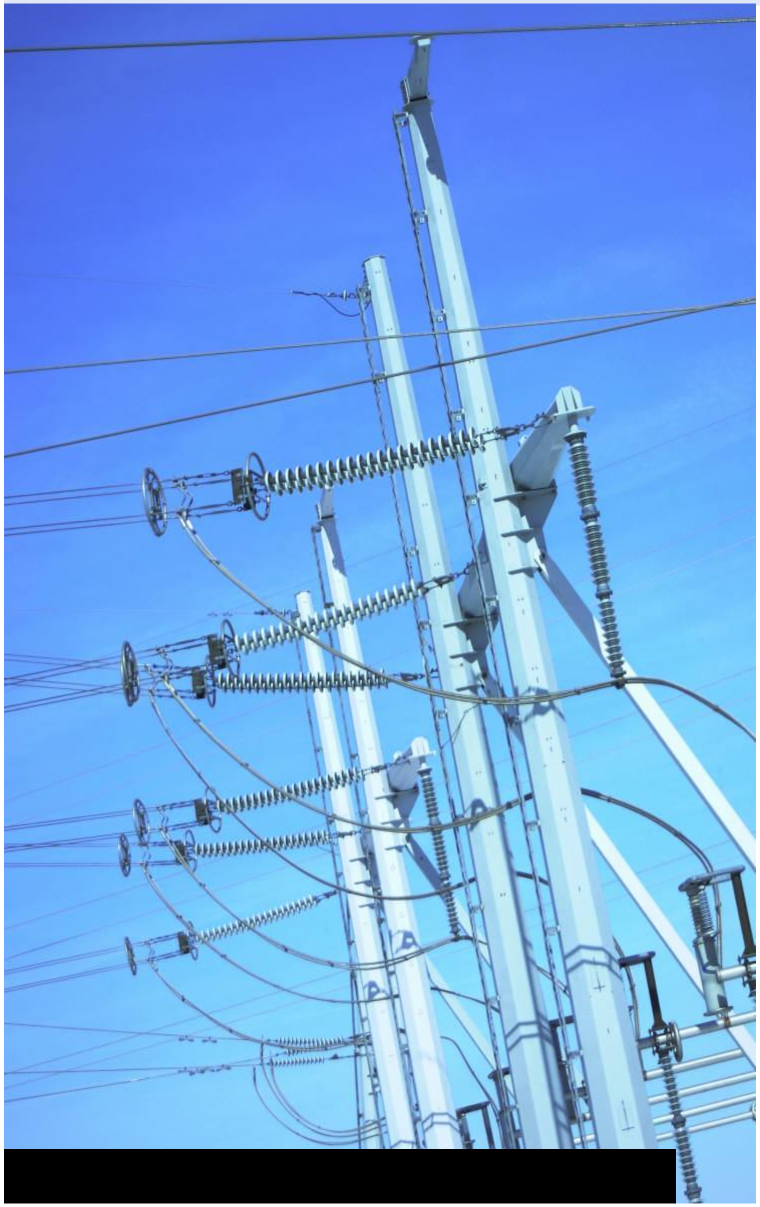
- CE, ME, EE graduates with 0-5 years experience and working on T-Line or Substation Design
- Experienced Design Engineers/Project Managers moving to utility industry
- GU Engineering undergraduates (juniors/seniors)

Instructors

Adjunct Faculty

- Experienced engineers/managers with utility and/or consulting background – four per course
- GU faculty from Civil Engineering, Mechanical Engineering, and Electrical Engineering Departments

Industry Need For T&D Engineers



- 1994-2004 Investment $2-3B/year
- 2004-2014 Investment $5-7B/year
- 2005 Federal Energy Legislation – NERC Compliance
- Blackouts and Shortages

  California 1996 – Seasonal Ratings

  2000-2001 – Energy Crisis (Transmission Constraints)

  August 13, 2003 – NE Blackout impacts 50M people

- NERC May 2001

  "The nation is at, or is fast approaching, a crisis stage with respect to reliability of transmission grids."

Power Industry

- Limited capital investment in lines and major equipment over last 25-30 years

- Overall power system in need of repairs and upgrades

- Several “transmission only” companies started in last 5-6 years

- Expected shortage of trained T&D professionals as “baby boomers” retire